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THE CALCULATION OF CHANGE-WHEELS FOR SCREW-CUTTING ON LATHES

D. DE VRIES

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THE CALCULATION OF

CHANGE-WHEELS

FOR

SCREW-CUTTING ON LATHES

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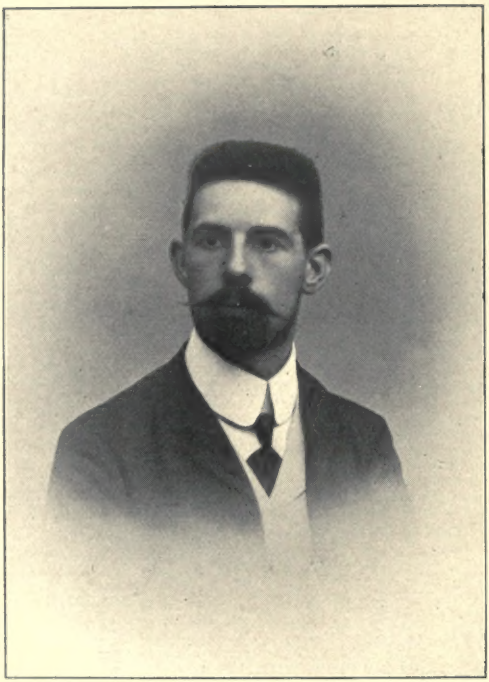

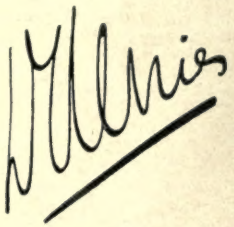

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THE CALCULATION OF

CHANGE-WHEELS

FOR

SCREW-CUTTING ON LATHES

A PRACTICAL MANUAL

FOR THE USE OF

MANUFACTURERS, STUDENTS AND LATHEMEN

BY

D. DE VRIES

WITH 46 ILLUSTRATIONS

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PREFACE

IT is a curious circumstance that the calculation of change-wheels for the cutting of different pitches of thread on a lathe, however simple such a calculation may be, is comparatively but little known, being, for the majority of those most closely interested in the subject, shrouded in mystery.

Many whose theoretical knowledge is quite sufficient to enable them to face the problem, have had so little practical experience in screw-cutting that they are unable to go deeply into the matter, and present, in a clear and simple manner, the different variations which may possibly occur.

The greater number of mechanics, even the younger ones, possess too slight a theoretical knowledge to permit of their building up a system by themselves.

There are, of course, mechanics who are quite capable of working out the necessary calculation, but so many of them — I speak from personal experience — regard their knowledge as more or less of a secret, and say, at any rate to themselves, "Why should I impart to others what has taken me so much trouble and cost me so much money to learn?"

The purpose of the present treatise is to enable any one, who is prepared to take the trouble to study it carefully to learn how to calculate change-wheels properly.

I have deemed it expedient, for the sake of those of my readers who have but a superficial knowledge of the lathe, to give a short description of this tool, in so far as it is connected with screw-cutting, to which I have added a description of the various types of thread to be met with, with the necessary tables appended, as also a number of practical hints, with reference to screw-cutting, together with the operations connected therewith.

I have purposely refrained from including a number of tables giving the change-wheels required for the various pitches of threads on different lathes, in place of which a large number of practical examples are given which cover every possible variation likely to be met with in practical work. Experience has taught me that the inclusion of such tables only leads to purely mechanical work demanding no effort of the mind, whereas, in each particular case, due consideration should be given to the special work in hand, so that in cases of exceptional difficulty, where one is obliged to set to work without the assistance of such tables, the manner of calculation may not be unfamiliar.

It is my earnest wish that the present work may prove useful not only to students, but also to those engaged in practical work.

D. DE VRIES.

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THE CALCULATION OF

CHANGE-WHEELS

FOR SCREW-CUTTING ON LATHES

CHAPTER I.

THE LATHE.

THREADS, both internal and external, can be obtained in two different ways, the simplest of which is to cut the thread by means of taps, dies and chasers. In the smaller sizes, the majority of internal threads are tapped, whilst external threads are cut with dies, but in the larger sizes too much material has to be removed. Tapping, however, is far more general than the use of dies, as in most cases, external threads can be obtained in another way, viz.: on the lathe, whilst internal threads can only be obtained on the lathe at considerable expense. Moreover, internal threads are to be found in a number of different places on the larger machine parts, and so it would be well-nigh impossible to put these pieces on the lathe for the purpose of cutting the threads. On the other hand, a bolt or screw-spindle, as a rule, can be set on the lathe, and threads may be cut by means of a common tool. It is just for this reason that, whilst a large number of 1 in. external threads are cut on the lathe, 1 in. threads in holes are, with but few exceptions, cut exclusively by tapping. The practice, however, of cutting internal threads of more than 2 in. diameter on the lathe, whenever the work-piece allows it, is becoming more and more general.

The object of the present work is to give a detailed description of the way in which it is possible to cut the various

threads on the lathe, and thus to answer, as fully as possible, the question: "How are the change-wheels to be calculated for screw-cutting on the lathe?"

In order that this work may also be of service to those who are not fully conversant with the lathe, the following points will be treated successively, viz.: the general construction of the lathe, more especially of those parts of the lathe used in screw-cutting; the theory of the calculation of change-wheels and screw-cutting in practice.

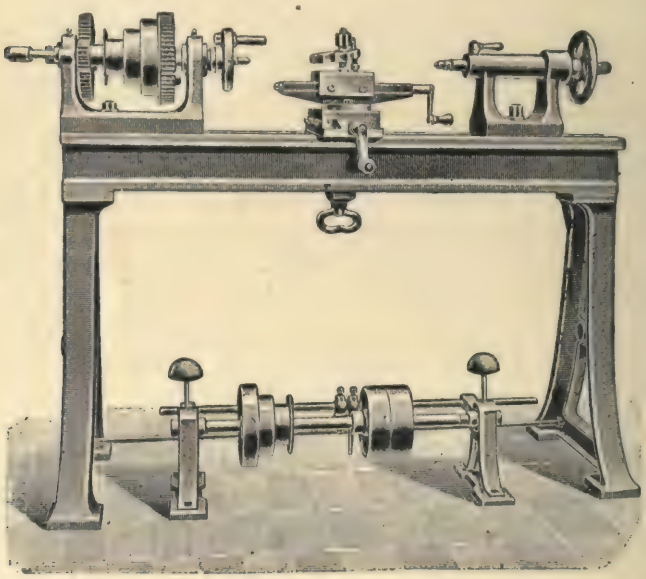


FIG. 1.

The lathe, as originally constructed, was not intended for screw-cutting. Fig. 1 shows a lathe as it was first constructed. On this lathe a rotary movement was imparted by means of a driving belt to the headstock and workpiece only, all other movements being executed by the operator himself.

Within a comparatively short time, however, more was demanded of this machine, larger pieces were required to be machined than was possible with direct belt drive, and the

double back gear was introduced ; it was desired to move the tool on the material automatically, and to obtain this, the rest was mounted on a carriage and moved by means of a leadscrew which motion was imparted by means of either a belt or a train of gears from the headstock. The introduction of a train of gears on the apron made it possible not only to move the carriage over the whole length of the bed for sliding, but also to move the rest automatically in a transverse direction over the carriage itself for surfacing.

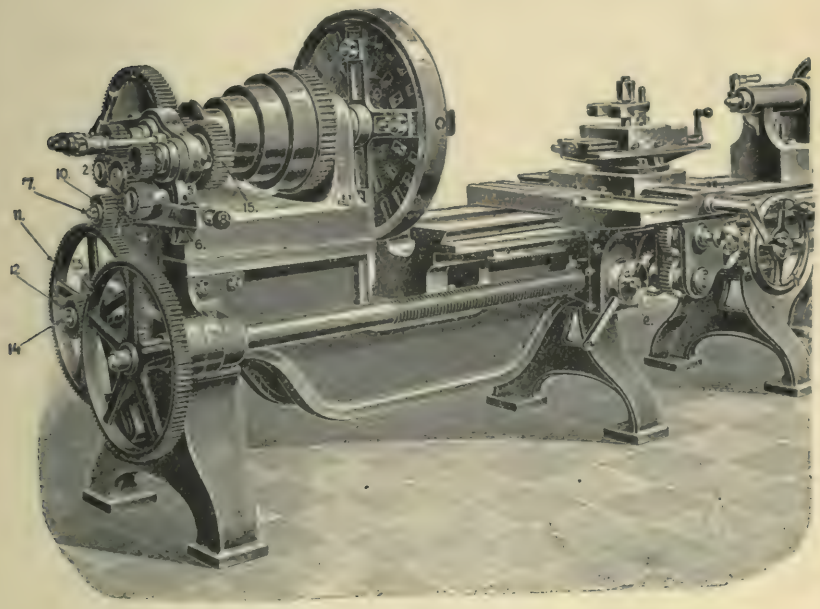


FIG. 2.

Finally, the leadscrew spindle, called for short the "leadscrew," was so arranged that by a set of gears of various diameters, a variable, but at the same time for each train of gears fixed ratio between the number of revolutions of the headstock, i.e. the workpiece, and the leadscrew was obtainable, thus making it possible to cut different pitches of threads on the lathe. Fig. 2 gives the general arrangement of such a lathe.

The leadscrew revolves in the leadscrew-nut, which is fixed to the apron, and, as this nut cannot revolve, it travels along the leadscrew, the carriage at the same time making a corresponding movement.

The movement of the carriage already causes a considerable pressure on the thread of the leadscrew and the nut, which is still increased by the cutting of the tool on the material, and, as a natural result, both the leadscrew and the nut are exposed to a certain amount of wear. This wear is further increased by swarf and chips falling on the leadscrew, and their getting between the nut and thread.

It is evident, as far as the leadscrew is concerned, that this wear will only affect that portion over which the nut travels on the thread. As the work on the lathe varies in length (but is as a rule considerably shorter than the maximum distance between the centres), the wear of the thread is greatest on those parts of the leadscrew where the nut moves, and after being in use for a certain time, it is impossible to prevent the leadscrew being scarcely worn at all at the end but considerably worn in the centre, and worn most of all close to the headstock. The wear of the nut, however, is fairly even.

The nut was formerly made solid, consequently it was impossible to repair the wear. It was soon seen, however, that it was preferable to have half nuts, so that not only can it now be repaired, but, by means of the lever *a* (Fig. 2), it can also be opened and closed.

This has led to the attainment of a number of advantages. First and foremost, the possibility of repairing the nut just referred to. A downward pressure of the lever *a* keeps both halves of the nut closed so as to grip the lead-screw. The two halves of the nut *b b* move in a vertical direction at the back of the piece *c*, and are provided with pins which fit in eccentric slots in the circular plate which revolves on point *e*. Fig. 3 shows these eccentric grooves in the plate. If the pins of the half nuts are shifted by moving the lever *a*, the half nuts travel the double distance A B (Fig. 3), viz.: the upper nut up and the lower one down, the half nuts being thus

entirely disengaged from the thread, causing the motion imparted to the carriage by the leadscrew to cease immediately.

In the earlier types of construction, with the solid nut, the carriage had to be moved by hand by means of a handle placed on a spindle in the apron, with a bevel gear on the other side of the spindle to which this handle was attached; this in its turn meshed with another bevel gear fixed on the hub of the nut. In this way the nut was made to revolve over the leadscrew and the carriage was moved over the bed. But it took far too long to move the carriage any distance at all over the bed, besides being very fatiguing work. The nut, being in halves, can no longer revolve, but it can be opened. A rack is to be found on the side of the bed in which a pinion meshes to which motion is imparted by the hand wheel *h* (Fig. 2), by means of which the carriage can be quickly disengaged from the leadscrew, and a quick and easy hand movement is secured.

FIG. 3.

Other advantages besides those enumerated here have been derived from the split nut. One great difficulty, however, still remains, viz., the different wear on a certain length of the leadscrew. If this happens to be more worn in the middle than at the ends, it is impossible to cut a true thread.

Now, in comparison to the work ordinarily performed on a lathe, but little screw-cutting is done. The greater part of the time the leadscrew is thus engaged for the feed motion of the carriage and for surfacing. For this reason, the movement imparted to the carriage for screw-cutting, has been separated from that for feed motion. A separate shaft, provided with a

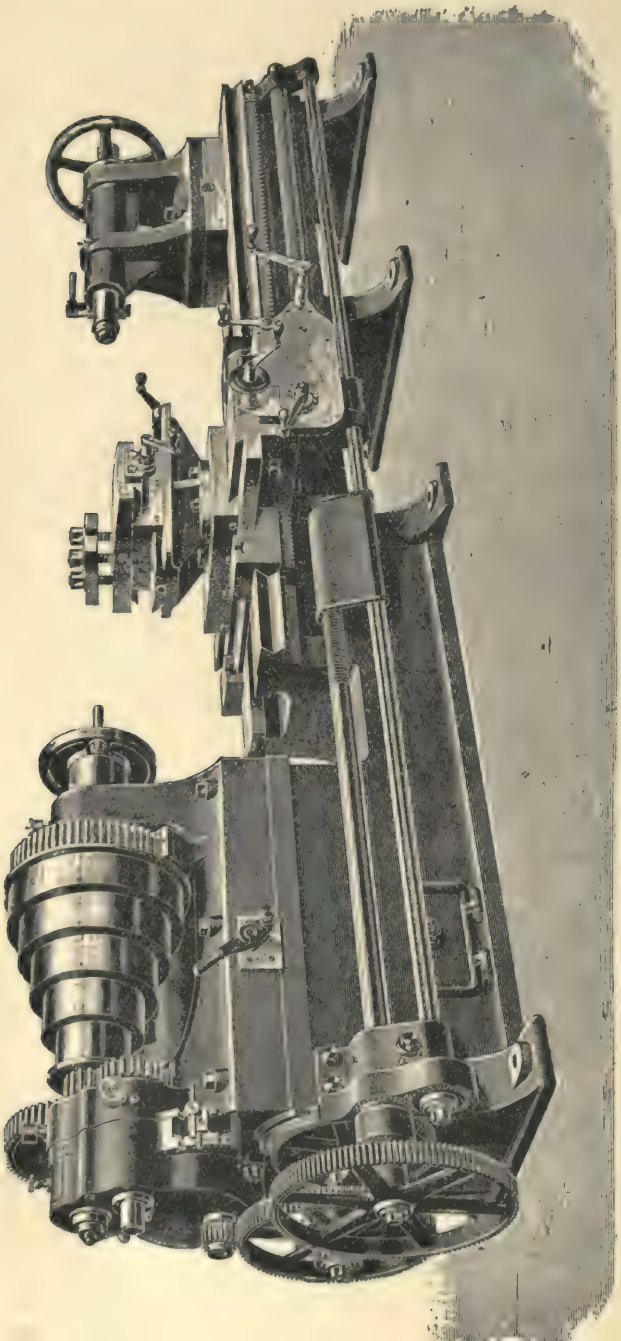


FIG. 4.

keyway, imparts motion to the pinion which meshes with the rack (Fig. 4), by means of bevel and spur-gears. The sliding movement of the carriage being accomplished in this manner, the leadscrew is only used for screw-cutting. In still later, and principally American constructions, the two shafts have finally been united in one, the leadscrew being now provided with a keyway ; for sliding and surfacing the leadscrew simply acts as driving shaft, the thread of the leadscrew being only used for screw-cutting, and so the same object is attained with one shaft as is obtained in Fig. 4 with two, viz., the thread of the leadscrew is used for screw-cutting only.

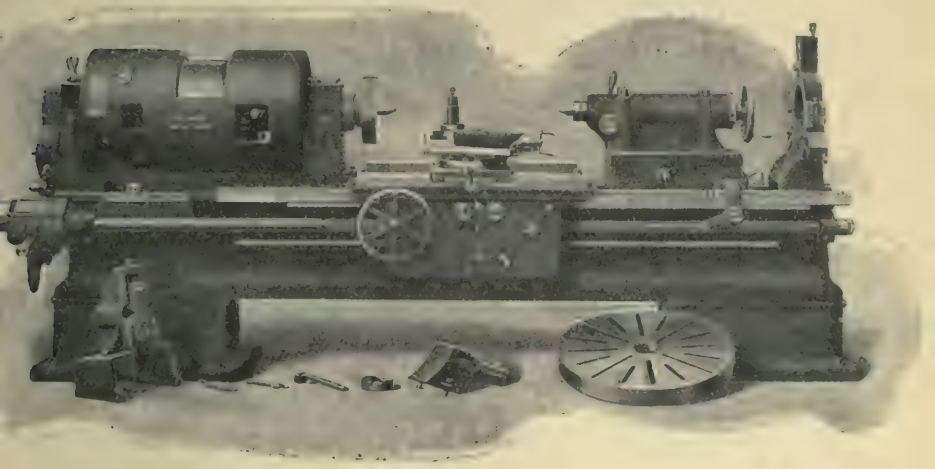


FIG. 5.

In Fig. 2 the gearing for the motion of the leadscrew from the head spindle is clearly visible. Wheel 1 is keyed to the head spindle ; rear wheels 2 and 3 run loose on studs fastened to the lever 4. By means of knob 8, this lever can be raised to hole 5 or lowered to hole 6. If the lever is placed in position 5, wheels 3 and 1 become engaged, and wheel 10 on spindle 7 revolves by means of wheel 9. Wheel 2 now runs to no purpose. If the lever is placed in position 6, wheels 2 and 1 become engaged, and wheel 3 is brought into play by means of wheel 2, thus causing wheel 3, as well as wheel 9 and

spindle 7 to rotate in an opposite direction. In the illustration the lever stands midway, so that wheel 1 engages neither of the wheels 2 or 3, consequently, although the lathe spindle rotates, the leadscrew is not rotating. Wheels 1, 2, 3 and 9 have the same number of teeth, so that the wheels on spindle 7 make precisely the same number of revolutions as the lathe spindle. Wheels 10, 11, 12 and 13 are the actual change-wheels, and can be easily mounted, dismounted, or changed. Wheels 11 and 12 rotate on a sleeve on spindle 14, and consequently make the same number of revolutions, so that wheel 12 transmits very slowly to wheel 13 the motion imparted to wheel 11. In the illustration the gearing between wheel 9 to the leadscrew is accomplished by 4 wheels—wheels 10 and 12 being the driving wheels, 11 and 13 those driven. It is evident that the motion of wheel 9 on spindle 7 is imparted but very slowly to the leadscrew, in the same ratio as the

FIG. 6.

product of the number of teeth on wheels 10 and 12 to the number of teeth on 11 and 13. Precisely the same is to be seen in Fig. 4. Wheel 13 can, however, be driven by means of a wheel engaging both wheels 10 and 13, without the intermediate wheels 11 and 12, thus serving as an idle wheel, in which case wheel 10 is the driving wheel and 13 the one driven. The ratio between the number of revolutions of the lathe-spindle and leadscrew is identical with the ratio between the number of teeth on wheels 10 and 13.

Wheels 11 and 12 are mounted on a sleeve running on stud 14. (See Fig. 6.)

This stud must be movable in accordance with the dimensions of the wheels, and is consequently placed in a casting called the shear or swingplate at the end of the lathe. This shear (Fig. 7), has two long slots, so that the stud can either be brought close to the leadscrew B, for small wheels,

or more to the rear for larger wheels, at will. In order to permit of working with five or six wheels, a second slot is to be found in the shear. This shear turns on the leadscrew B, and is held in position by means of the two bolts to be seen in the circular slots. When the intermediate wheels have been accurately set in the wheel on the leadscrew, the shear, which was first lowered to its full extent, is raised till the intermediate wheel engages the upper wheel properly, after which the shear is fastened.

Fig. 5 shows an American type of lathe, on which it is not necessary to change the wheels for different pitches of

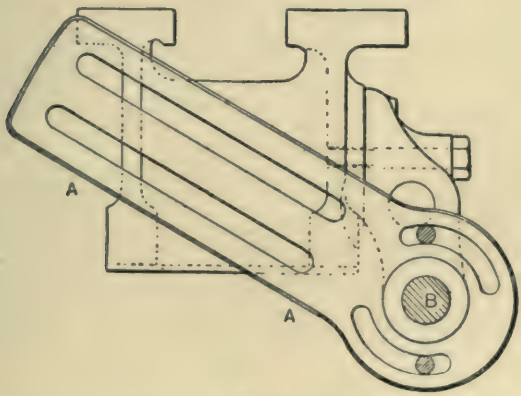


FIG. 7.

threads. By means of a cone-gear to be found under the headstock and at the left-hand side of same, the ratio of speed between the lathe-spindle and the leadscrew can be varied by the simple movement of a lever. The necessity of calculating the change-wheels is done away with, all that is required being the placing of two levers in a certain position indicated in the table. The manner in which this result is attained will be further described in Chapter III.

CHAPTER II.

THE CALCULATION OF CHANGE-WHEELS.

(*a*) *Systems.*

IN the calculation of change-wheels for screw-cutting on the lathe there is one difficulty, and that is, the difference between the English and metric system of measurements. It is not insurmountable, but it does not render the task any easier, and has been the cause of a considerable amount of trouble.

In the calculation of change-wheels it is a matter of indifference whether a right- or left-handed screw is to be cut, what form the thread has to take, whether the thread is internal or external, or, finally, the exact internal or external diameter of the thread. The one essential question to be answered is: How many threads are required for a certain unit of length?

For this purpose two units exist; 1st, the inch; 2nd, the centimetre.

For both these units of length the number of revolutions of the thread are termed "number of threads."

The length of a single thread is spoken of as "pitch."

The number of threads is thus determined by the number of revolutions per unit of length.

If the pitch is indicated with the inch as the unit of length, we speak of "English thread." If the pitch is indicated with the centimetre as unit of length, it is called a "metric thread." Both, however, have a system, which is further treated of in Chapter III., but which, as such, has nothing at all to do with the calculation of the change-wheels.

If but one of these two units, either the inch or the centimetre, were exclusively adopted as the standard unit, then the difficulty referred to at the beginning of this chapter would

entirely disappear. But the inch and the centimetre are employed together ; and not only that, but there is also a lack of uniformity with regard to the leadscrew; one maker cutting the leadscrew according to the English, and another according to the metric system. English and American lathes usually have a leadscrew cut according to the English system ; French and Swiss makers cut it almost exclusively according to the metric system, whilst German manufacturers employ both systems, though the preference is given to the English.

Four variations are thus possible :—

1. A metric thread to be cut on a lathe with metric leadscrew.
2. An English thread to be cut on a lathe with English leadscrew.
3. An English thread to be cut on a lathe with metric leadscrew.
4. A metric thread to be cut on a lathe with English leadscrew.

Briefly summarized :—

To cut: 1. Metric on metric.
2. English on English.
3. English on metric.
4. Metric on English.

If one desires, once and for all, to be able to calculate the change-wheels for every variety of pitch, it is imperative to know these four varieties thoroughly, as they can occur intermingled.

1st Axiom.—The number of threads is to be determined by the pitch of the leadscrew and the ratio of the number of revolutions of the lathe spindle to that of the leadscrew.

This axiom holds good for all four cases.

The ratio of the number of revolutions of the lathe-spindle to that of the lead-screw is obtained by means of wheels (change-wheels).

When the spindle of the lathe has completed one revolution, then the work on the lathe will have also completed one revolution.

If the number of revolutions of the lathe-spindle and leadscrew are the same, so that the leadscrew has also completed one revolution, then the carriage has moved a distance during this one revolution equivalent to one thread of the leadscrew. If a tool has been placed in the toolholder, so that it can cut the work-piece, then precisely the same pitch will have been cut on the work-piece as that on the leadscrew. With an equal number of revolutions of the lathe-spindle and the leadscrew, the thread cut on the work-piece will have the same pitch as the leadscrew.

If the lathe-spindle has completed one full revolution, but the leadscrew on the other hand only half a revolution, then the carriage, and with it the tool, will have moved in a straight line over a length equal to half a pitch of the leadscrew. It is thus only when the lathe-spindle has made two revolutions that the leadscrew will have completed one full revolution; two threads are now to be found on the work-piece over a length equal to one pitch of the leadscrew. The ratio of the number of revolutions of the spindle to that of the leadscrew was 2 : 1; the ratio of the number of threads per unit of length of the work-piece to that of the leadscrew was also 2 : 1. Hence it follows :—

2nd Axiom.—The ratio of the number of revolutions of the lathe-spindle to that of the leadscrew is the same as the proportion of the pitch of the thread to be cut to that of the leadscrew.

Axiom 2 is also applicable to all four cases.

For example, the leadscrew of a lathe has a pitch of one thread to the inch. It is required to cut two threads to the inch. The proportion of the pitch to be cut to that of the leadscrew is thus 2 : 1. According to axiom 2 the ratio of the number of revolutions of the lathe-spindle to that of the leadscrew must also be 2 : 1.

The leadscrew has thus to complete one revolution to two of the lathe-spindle. The leadscrew receives its motion from the lathe-spindle, so that the rotation of the leadscrew must be retarded accordingly. The rotation of the lathe-spindle is transmitted to the leadscrew by wheels. The pro-

portion of the number of teeth on wheel 10 (see Fig. 2), to those on wheel 13 on the leadscrew must thus be in inverse proportion to the ratio between the number of revolutions of the lathe-spindle and the leadscrew, which, in the example given, must be 2 : 1 ; the ratio of the wheels 10 and 13 thus becomes 1 : 2. If then a wheel with 50 teeth be on the sleeve of spindle 7, and one with 100 teeth on the leadscrew, with any desired idle wheel, a screw of 2 threads to the inch or ½-inch pitch will be obtained on the work-piece with a leadscrew having one-inch pitch. From this we arrive at what is again applicable to all four cases :—

3rd Axiom.—The proportion of the number of the threads to be cut to those in the leadscrew is in inverse ratio to the proportion of the number of teeth on the wheel on the lathe-spindle to the number of teeth on the wheel of the lead-screw, or in fractional form—

$$\frac{\text{Number of threads to be cut}}{\text{Number of threads in the leadscrew}} = \frac{\text{No. of teeth on the leadscrew wheel}}{\text{No. of teeth on the lathe-spindle wheel}}$$

In this manner the calculation of the change-wheels for screw-cutting is reduced to the working out of a simple fraction—the number of threads to be cut being the numerator, those in the leadscrew being the denominator, or, if it is desired to express the fraction in the same manner as the wheels, i.e. the number of teeth on the lathe-spindle wheel on top as numerator, that of the leadscrew underneath as denominator, it is just the reverse. The number of threads in the leadscrew will then represent the value of the numerator, those of the thread to be cut representing the denominator. As the pitch of the leadscrew on a certain lathe is always the same, it follows that the value of the numerator is always *constant*.

We must here call especial attention to a misunderstanding which so often occurs in connection with the question as to whether the number of threads in the leadscrew must form

the numerator or the denominator. A practical man can generally tell fairly well which wheels have to be placed on top and which underneath, but still, when the pitch of the thread to be cut closely approximates that of the leadscrew, mistakes can sometimes be made.

The screw may be denoted by the number of threads per unit of length, in which case the number of threads in the leadscrew is the numerator of the fraction.

The screw may also be denoted by the length of one pitch of the screw ; in this case the length of pitch of the screw to be cut will be the numerator, the length of pitch of the lead-screw being the denominator of the fraction, the numerator of which will indicate the number of teeth on the lathe-spindle wheel, the denominator indicating the number of teeth of the wheel on the leadscrew.

Should the number of threads of the screw to be cut be a multiple of those in the leadscrew, one is naturally inclined to express it in number of threads per unit ; for example, 4 threads per inch to be cut on a lathe with a leadscrew of 1 thread per inch ; should it not be a multiple, as for example, each thread having a length of 7 mm., one is then inclined to denote it by the pitch. If, in both instances, the number of threads in the leadscrew be 1 per inch, the fraction in the first instance will be—

$$\frac{\text{Number of threads in the leadscrew}}{\text{Number of threads to be cut}} = \frac{1}{4} = \frac{\text{driving wheel}}{\text{wheel to be driven}}$$

In the second instance, in which the pitch of the screw to be cut must be 7 mm., the number of the threads to be cut per unit is itself a fraction, viz. : $\frac{25 \cdot 4}{7}$, the fraction thus being $\frac{1}{\frac{25 \cdot 4}{7}} = \frac{7}{25 \cdot 4}$, 7 being the length in mm. of the pitch of the screw to be cut, 25·4 the length in mm. of the pitch of the lead-screw, so that, in this case, the length of pitch of the screw to be cut can at once be placed in the numerator for the driving wheel, the length of pitch of the leadscrew being

placed in the denominator for the wheel to be driven. In actual calculation the foregoing examples must be carefully distinguished one from the other.

(*b*) *What Change-wheels are to be found on a Lathe.*

This question presents itself each time change-wheels have to be calculated, because the fraction which is formed by the thread to be cut and the leadscrew, must be changed into one formed from the wheels to be found on the lathe. These wheels should have such a number of teeth as will, within certain limits, include the indivisible factors, viz.: 2, 3, 5, 7, 11, 13, 17, 19, 23, etc. Some makers supply these wheels in a progression of 5, others with another progression. The following set of change-wheels is, or should be provided with every lathe, viz.:—

$15 = 3 \times 5$	$60 = 2 \times 2 \times 3 \times 5$
$20 = 2 \times 2 \times 5$	$65 = 5 \times 13$
$25 = 5 \times 5$	$75 = 3 \times 5 \times 5$
$30 = 2 \times 3 \times 5$	$85 = 5 \times 17$
$35 = 5 \times 7$	$95 = 5 \times 19$
$40 = 2 \times 2 \times 2 \times 5$	$100 = 2 \times 2 \times 5 \times 5$
$45 = 3 \times 3 \times 5$	$105 = 3 \times 5 \times 7$
$50 = 2 \times 5 \times 5$	$115 = 5 \times 23$
$55 = 5 \times 11$	$125 = 5 \times 5 \times 5$

or

$16 = 2 \times 2 \times 2 \times 2$	$42 = 2 \times 3 \times 7$
$18 = 2 \times 3 \times 3$	$44 = 4 \times 11$
$20 = 2 \times 2 \times 5$	$56 = 2 \times 2 \times 2 \times 7$
$21 = 3 \times 7$	$60 = 2 \times 2 \times 3 \times 5$
$22 = 2 \times 11$	$66 = 2 \times 3 \times 11$
$26 = 2 \times 13$	$78 = 2 \times 3 \times 13$
$28 = 2 \times 2 \times 7$	$88 = 2 \times 2 \times 2 \times 11$
$34 = 2 \times 17$	$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$38 = 2 \times 19$	$108 = 2 \times 2 \times 3 \times 3 \times 3$

One of the two foregoing sets is generally provided with

the lathe. English lathes usually have a set of 22 wheels some of which have the same number of teeth.

It will be clear from what has been said, thus far, that the easiest thread to be cut on a lathe, i.e. the thread causing the least trouble in the calculation of the change-wheels, is that having the same system as the leadscrew. This will be the case with the 1st and 2nd cases referred to on page 11.

(*c*) *The Cutting of Metric Threads on a Lathe with Metric Leadscrew.*

Take the case of a lathe with a leadscrew having 1 cm. (10 mm.) pitch.

It is required to cut 4 threads per cm.

$$\frac{\text{No. of teeth on driving wheel}}{\text{No. of teeth on wheel to be driven}} = \frac{\text{No. of threads in the leadscrew}}{\text{No. of threads to be cut.}}$$

$$= \frac{1}{4} = \frac{25}{100} = \begin{matrix}\text{gear-wheel 10.} \\ \text{gear-wheel on lead-screw.}\end{matrix} \Big\} \text{ See Fig. 2.}$$

It is required to cut 7 threads per cm.

$$\frac{\text{No. of threads in the leadscrew}}{\text{No. of threads to be cut}} = \frac{1}{7} = \frac{15}{105} \begin{matrix}= \text{driving wheel.} \\ = \text{wheel to be driven.}\end{matrix}$$

To cut $1\frac{1}{2}$ thread per cm.

$$\frac{\text{No. of threads in the leadscrew}}{\text{No. of threads to be cut}} = \frac{1}{1{\cdot}5} = \frac{50}{75} \text{ or } \begin{matrix}60 = \text{driving wheel.} \\ 90 = \text{wheel to be driven.}\end{matrix}$$

To cut 3 threads per cm.

$$\frac{\text{No. of threads in the leadscrew}}{\text{No. of threads to be cut}} = \frac{1}{3} = \frac{30}{90} \begin{matrix}= \text{driving wheel.} \\ = \text{wheel to be driven.}\end{matrix}$$

To cut 5 threads per cm.

$$\frac{\text{No. of threads in the leadscrew}}{\text{No. of threads to be cut}} = \frac{1}{5} = \frac{25}{125} \begin{matrix}= \text{driving wheel.} \\ = \text{wheel to be driven.}\end{matrix}$$

In the last example it is also possible to say, a pitch of 2 mm., in which case the fraction will be :—

$$\frac{\text{Pitch in mm. to be cut}}{\text{Pitch in mm. of leadscrew}} = \frac{2}{10} = \frac{25}{125} \begin{matrix}= \text{driving wheel.} \\ = \text{wheel to be driven.}\end{matrix}$$

In both cases the result will naturally be the same.

To cut a pitch of 7 mm.

$$\frac{\text{Pitch in mm. to be cut}}{\text{Pitch in mm. of leadscrew}} = \frac{7}{10} = \frac{70}{100} = \frac{\text{driving wheel.}}{\text{wheel to be driven.}}$$

To cut a pitch of $5\frac{1}{2}$ mm.

$$\frac{\text{Pitch in mm. to be cut}}{\text{Pitch in mm. of leadscrew}} = \frac{5.5}{10} = \frac{55}{100} = \frac{\text{driving wheel.}}{\text{wheel to be driven.}}$$

To cut 7 threads per 22 mm.

Denoted in pitch = a pitch of $\frac{22}{7}$ mm.

$$\frac{\text{Pitch in mm. to be cut}}{\text{Pitch in mm. of leadscrew}} = \frac{{}^{22}/_{7}}{10} = \frac{22}{70} = \frac{\text{driving wheel.}}{\text{wheel to be driven.}}$$

FIG. 8.

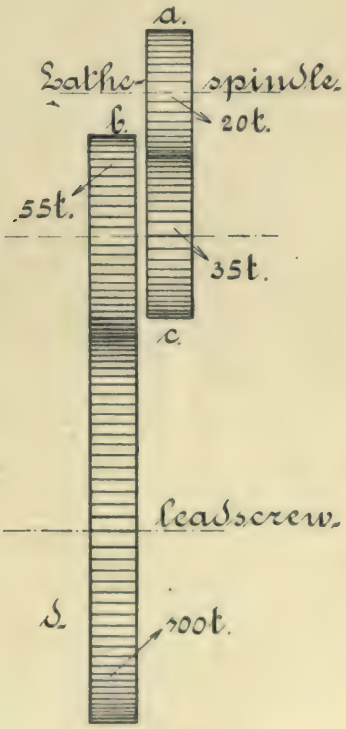


FIG. 9.

So far it has always been possible to work with a single train of wheels with any desired idle wheel. Fig. 8 shows a single train.

In the set of wheels to be found on the lathe, wheels with either 22 or 70 teeth, as presumed were employed for the preceding examples, were not included. A compound train is now used.

$$\frac{22}{70} = \frac{2 \times 11}{7 \times 10} = \frac{20 \times 55}{35 \times 100} = \begin{matrix}\text{driving wheels.} \\ \text{wheels to be driven.}\end{matrix}$$

Fig. 9 shows this compound train.

a and *b* are the *drivers*, *c* and *d* those *driven*. The fixing up of the wheels will thus be

$$\frac{55 \times 20}{100 \times 35}.$$

The wheels in the numerator, as well as those in the denominator, can be interchanged; *a* may thus be put in place of *b*, or *c* in place of *d*, or both may be changed; but interchanging of a driver with one to be driven may never take place, as this would alter the value of the fraction and an entirely different thread would be obtained.

It is always advisable to try to get the smallest of the drivers on the lathe-spindle, and the largest to be driven on the leadscrew, in order to obtain as rational a gearing as possible.

To cut 11 threads per 14 mm. The pitch is thus 14/11 mm. Pitch on leadscrew 10 mm.

$$\text{Solution:} \quad \frac{14/11}{10} = \frac{14}{11 \times 10} = \frac{2 \times 7}{11 \times 10} = \frac{20 \times 35}{55 \times 100}.$$

To cut $3\frac{1}{2}$ thread per 40 mm. The pitch is thus 40/3·5 mm.

$$\text{Solution:} \quad \frac{40/3 \cdot 5}{10} = \frac{40}{35 \times 10} = \frac{4 \times 10}{5 \times 7} = \frac{20 \times 100}{50 \times 35}.$$

To cut 4 threads on 15 mm. The pitch is thus 15/4 mm.

$$\text{Solution:} \quad \frac{15/4}{10} = \frac{15}{4 \times 10} = \frac{3 \times 5}{4 \times 10} = \frac{30 \times 50}{40 \times 100}.$$

Should the lathe have another pitch than 1 cm., this will only necessitate a change in the constant of the leadscrew in the fraction.

The following are a few examples with solutions, dealing with different leadscrews :—

To cut 9 threads per 16 mm.; leadscrew 2 threads per 1 cm. The pitch of the thread to be cut in 16/9 mm. The pitch of the leadscrew is 5 mm.

Solution: $\frac{16/9}{5} = \frac{2\times 8}{9\times 5} = \frac{20\times 40}{45\times 50}$ = in case these wheels are too small $\frac{20\times 80}{45\times 100}$.

To cut a pitch of 3 mm. Pitch of leadscrew being 7·5 mm.

Solution: $\frac{3}{7\cdot 5} = \frac{30}{75}$.

To cut 8 threads per 13 mm. Pitch of leadscrew, 7·5 mm.

Solution: $\frac{13/8}{7\cdot 5} = \frac{13}{8\times 7\cdot 5} = \frac{2\times 6\cdot 5}{8\times 7\cdot 5} = \frac{20\times 65}{80\times 75}$.

In both the foregoing examples, a wheel with 75 teeth appears among the wheels driven, but is not included in the specification given on page 15. With a leadscrew having a pitch of 7·5 mm. a wheel with 75 teeth will repeatedly occur; in such a case the manufacturer will be certain to supply a wheel with 75 teeth.

To cut a pitch of 20 mm. Leadscrew pitch 25 mm.

Solution: $\frac{20}{25} = \frac{100}{125}$.

To cut 3 threads per 20 mm. Leadscrew pitch 25 mm.

Solution: $\frac{20/3}{25} = \frac{20}{3\times 25} = \frac{2\times 10}{3\times 25} = \frac{40\times 50}{60\times 125}$.

To cut a pitch of 37·5 mm. Leadscrew pitch 25 mm.

Solution: $\frac{37\cdot 5}{25} = \frac{15\times 25}{10\times 25} = \frac{30\times 100}{40\times 50}$.

To cut a pitch of 76 mm. Leadscrew pitch 25 mm.

Solution: $\frac{76}{25} = \frac{4\times 19}{2\cdot 5\times 10} = \frac{40\times 95}{25\times 50} = \frac{80\times 95}{25\times 100}$.

(*d*) *The Cutting of English Threads on a Lathe with English Leadscrew.*

In principle, this second case resembles the first. The system of the leadscrew and the thread to be cut is the same.

Most lathes have a leadscrew with $\frac{1}{2}$ in. pitch, thus 2 threads per inch. Heavy lathes have a leadscrew with 1 in. pitch, the smaller sizes $\frac{1}{4}$ in., or 4 threads per inch, whilst in exceptional cases $2\frac{1}{2}$ threads per inch are to be found. Given a certain pitch, the fraction can then be determined without any difficulty.

Should the screw be denoted in a certain number of threads per inch, the number of threads per inch of the leadscrew is placed in the numerator, the number of threads per inch to be cut in the denominator. Should the screw be denoted in the length of the pitch, then the length in inches of the pitch to be cut is placed in the numerator, the length in inches of the pitch of the leadscrew being placed in the denominator.

In practice the majority of threads are cut according to the Whitworth system (see page 57), for which reason we shall first of all give a number of problems with solutions for this thread.

To cut $\frac{3}{8}$ in. Whitworth thread. Leadscrew 2 threads per inch. $\frac{3}{8}$ in. Whitworth thread = 16 threads per inch.

Solution : $$\frac{\text{No. of threads in leadscrew per inch}}{\text{No. of threads to be cut per inch}}$$

$$= \frac{2}{16} = \frac{25 \times 1}{16 \times 12{\cdot}5} = \frac{25 \times 50}{80 \times 125}.$$

To cut $1\frac{1}{8}$ in. Whitworth thread. Leadscrew 2 threads per inch. $1\frac{1}{8}$ in. Whitworth thread = 7 threads per inch.

Solution : $$\frac{2}{7} = \frac{2 \times 1}{3{\cdot}5 \times 2} = \frac{20 \times 40}{35 \times 80}.$$

To cut 2 in. Whitworth thread. Leadscrew 2 threads per inch. 2 in. Whitworth thread = $4\frac{1}{2}$ thread per inch.

Solution: $$\frac{2}{4\cdot5} = \frac{2 \times 10}{5 \times 9} = \frac{40 \times 50}{45 \times 100}.$$

To cut 3 in. Whitworth thread. Leadscrew 2 threads per inch. 3 in. Whitworth thread = $3\frac{1}{2}$ thread per inch.

Solution: $$\frac{2}{3\cdot5} = \frac{40}{70}.$$

To cut $1\frac{1}{4}$ in. Whitworth thread. Leadscrew 4 threads per inch. $1\frac{1}{4}$ in. Whitworth thread = 7 threads per inch.

Solution: $$\frac{4}{7} = \frac{40}{70}.$$

To cut $1\frac{1}{2}$ in. gas thread. Leadscrew $2\frac{1}{2}$ thread per inch. $1\frac{1}{2}$ in. gas thread = 11 threads per inch.

Solution: $$\frac{2\frac{1}{2}}{11} = \frac{25}{110} = \frac{20 \times 50}{55 \times 80}.$$

To cut $2\frac{1}{4}$ thread per inch. Leadscrew 2 threads per inch.

Solution: $$\frac{2}{2\frac{1}{4}} = \frac{2 \times 4}{9} = \frac{2 \times 4}{3 \times 3} = \frac{20 \times 60}{30 \times 45}.$$

To cut $\frac{7}{8}$ thread per inch (*not a $\frac{7}{8}$ inch pitch*). Leadscrew 2 threads per inch.

Solution: $$\frac{2}{\frac{7}{8}} = \frac{2 \times 8}{7} = \frac{80}{35} = \frac{40 \times 100}{25 \times 70}.$$

To cut $2\frac{3}{4}$ thread per inch. Leadscrew $2\frac{1}{2}$ thread per inch.

Solution: $$\frac{2\frac{1}{2}}{2\frac{3}{4}} = \frac{10}{11} = \frac{50}{55}.$$

In the following examples, the *length* of pitch is given. The pitch of the leadscrew will consequently appear in the denominator.

To cut a $\frac{3}{4}$ in. pitch. Leadscrew 2 threads per inch = $\frac{1}{2}$ in. pitch.

Solution: $$\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2} = \frac{75}{50}.$$

To cut a $\frac{15}{16}$ in. pitch. Leadscrew $2\frac{1}{2}$ threads per inch $= \frac{1}{2\frac{1}{2}}$ in. pitch.

Solution: $$\frac{\frac{15}{16}}{\frac{1}{2\frac{1}{2}}} = \frac{\frac{15}{16} \times 2\frac{1}{2}}{1} = \frac{15 \times 2\frac{1}{2}}{16 \times 1} = \frac{75 \times 100}{40 \times 80}.$$

To cut 19 threads on 11·5 in. Leadscrew $2\frac{1}{2}$ threads per inch. The pitch to be cut $= \frac{11 \cdot 5}{19}$ in. The leadscrew pitch $\frac{1}{2\frac{1}{2}}$ in.

Solution: $$\frac{\frac{11 \cdot 5}{19}}{\frac{1}{2}} = \frac{11 \cdot 5 \times 2 \cdot 5}{19} = \frac{115 \times 125}{95 \times 100}.$$

To cut a pitch of $4\frac{7}{8}$ in. Leadscrew 1 pitch per inch.

Solution: $$\frac{4\frac{7}{8}}{1} = \frac{39}{8} = \frac{3 \times 13}{2 \times 4} = \frac{65 \times 75}{25 \times 40}.$$

To cut a $\frac{13}{32}$ in. pitch. Leadscrew 2 threads per inch.

Solution: $$\frac{\frac{13}{32}}{\frac{1}{2}} = \frac{13 \times 2}{32} = \frac{2 \times 13}{2 \times 16} = \frac{65}{80}.$$

To cut 9 threads per $5\frac{15}{16}$ in. Leadscrew $2\frac{1}{2}$ thread per inch. The pitch to be cut $= \frac{5\frac{15}{16}}{9}$ in. The pitch of the leadscrew $= \frac{1}{2\frac{1}{2}}$ in.

Solution: $$\frac{\frac{5\frac{15}{16}}{9}}{\frac{1}{2\frac{1}{2}}} = \frac{5\frac{15}{16} \times 2\frac{1}{2}}{9} = \frac{95 \times 2 \cdot 5}{9 \times 16} = \frac{125 \times 95}{80 \times 90}.$$

In the foregoing examples practically every case which is likely to occur, has been treated.

(*e*) *To cut English Threads on a Lathe with Metric Leadscrew.*

In the first and second cases considered, the system of the thread to be cut and that of the leadscrew were identical, viz., in the first case according to metric measurement, in the second, according to the English measurement.

In the third case, however, the system of the thread to be cut and that of the leadscrew are dissimilar. The leadscrew is divided per cm. = 10 mm., or some part or multiple thereof, the screw to be cut being divided per inch = 25·4 mm., or some part or multiple thereof.

In the third case to be considered, this number 25·4 will consequently appear regularly either in the numerator or the denominator, and will invariably produce a fraction which, with one exception, cannot be resolved into whole numbers.

An equivalent must therefore be found, by means of which it will be possible to form a divisible number from the numerator and denominator of the fraction.

This equivalent is to be found as follows: $6\frac{1}{2}$ in. = 16·509675 cm.; taking for granted that $6\frac{1}{2}$ in. = 16·5 cm., there is then a discrepancy of 0·09675 mm. per 165 mm. of length, or rather less than 0·06 per cent., a difference of practically no importance whatever.

If the number of threads to be cut be expressed in a certain number per 6·5 in., and the number of threads of the leadscrew be also expressed in a certain number per 6·5 in. or 16·5 cm., the result will be an equivalent which can be made use of.

As reference is here made to a certain number of threads per unit of length, in this case, 6·5 in. or 16·5 cm., the numbers of threads of the leadscrew will appear in the numerator, the number of threads to be cut in the denominator.

The following comparison can thus be formulated—

$$\frac{\text{No of threads in leadscrew per 16·5 cm.}}{\text{No. of threads to be cut per 6·5 in.}} = \frac{\text{drivers}}{\text{wheels to be driven}}$$

As the number of threads in the leadscrew remains invariable for the same lathe, the numerator is consequently a constant factor for a certain lathe.

Should the leadscrew have a 1 cm. pitch, the leadscrew will then have 16·5 threads per 16·5 cm., and the constant factor of the numerator will be 16·5, whilst, at the same time, 6·5 is to be found as a constant factor in the denominator, and must constantly be multiplied by the number which expresses the number of threads to be cut per inch. If both these constant factors be multiplied by 10, the number 165 will always appear in the numerator and the number 65 in the denominator, in this way—

constant factor of numerator . 165

" " denominator No. of threads to be cut per in. × 65

or $$\frac{11 \times 15}{\text{threads per in.} \times 65}.$$

The equivalent is now complete ; by replacing *threads per inch* in the denominator by the actual number, a fraction is obtained which will permit of the calculation of the wheels.

In the examples which follow, every possible variation has been carefully worked out, from the simplest to the most intricate.

To cut 6 threads per inch. Leadscrew 10 mm. pitch.

$$\text{Numerator} = 11 \times 15$$
$$\text{Denominator} = \text{No. of threads per inch} \times 65 = \frac{11 \times 15}{6 \times 65}$$

$$= \frac{55 \times 75}{150 \times 65} = \frac{50 \times 55}{65 \times 100}.$$

To cut 4 threads per inch. Leadscrew 10 mm. pitch.

$$\text{Solution : } \frac{11 \times 15}{4 \times 65} = \frac{11 \times 75}{20 \times 65} = \frac{55 \times 75}{65 \times 100}.$$

To cut $2\frac{1}{4}$ threads per inch. Leadscrew 10 mm. pitch.

$$\text{Solution : } \frac{11 \times 15}{\frac{9}{4} \times 65} = \frac{4 \times 11 \times 15}{9 \times 65} = \frac{2 \times 2 \times 3 \times 5 \times 11}{3 \times 3 \times 5 \times 13}$$

$$= \frac{12 \times 55}{45 \times 13} = \frac{55 \times 60}{45 \times 65}.$$

To cut $5\frac{1}{2}$ threads per inch. Leadscrew 10 mm. pitch.

Solution: $\frac{11 \times 15}{5 \cdot 5 \times 65} = \frac{11 \times 30}{11 \times 65} = \frac{30}{65}$.

To cut 1 in. Whitworth-thread = 8 threads per inch. Leadscrew 5 mm. pitch.

In this case the leadscrew has 2 threads per cm. Consequently. for this particular lathe, the numerator is $2 \times 165 = 330$ or 11×30.

Solution: $\frac{11 \times 30}{8 \times 65} = \frac{60 \times 55}{65 \times 80}$.

To cut $\frac{1}{4}$ in. gas thread = 14 threads per inch. Leadscrew 5 mm. pitch.

Solution: $\frac{11 \times 30}{14 \times 65} = \frac{30 \times 55}{65 \times 70}$.

To cut $\frac{1}{4}$ in. Whitworth-thread = 20 threads per inch. Leadscrew 5 mm. pitch.

Solution: $\frac{11 \times 30}{20 \times 15} = \frac{30 \times 55}{65 \times 100}$.

To cut 1 in. gas thread = 11 threads per inch. Leadscrew 6 mm. pitch. No. of threads in leadscrew per cm., $\frac{10}{6}$.

Solution: $\frac{\frac{10}{6} \times 11 \times 15}{11 \times 65} = \frac{\frac{10}{6} \times 15}{65} = \frac{10 \times 15}{6 \times 65} = \frac{25}{65}$.

To cut 36 threads per inch. Leadscrew 4 mm. pitch. No. of threads in the leadscrew per cm., $\frac{10}{4}$ or $2 \cdot 5$.

Solution: $\frac{2 \cdot 5 \times 11 \times 15}{36 \times 65} = \frac{11 \times 12 \cdot 5}{12 \times 65} = \frac{25 \times 55}{65 \times 120}$.

To cut $\frac{7}{8}$ thread per inch. Leadscrew 10 mm. pitch.

Solution: $\frac{11 \times 15}{\frac{7}{8} \times 65} = \frac{8 \times 11 \times 15}{7 \times 65} = \frac{11 \times 120}{7 \times 65} = \frac{55 \times 120}{35 \times 65}$.

To cut a $\frac{7}{8}$ in. pitch. Leadscrew 10 mm. pitch. No. of threads per inch $\frac{1}{\frac{7}{8}} = \frac{8}{7}$.

Solution: $$\frac{11 \times 15}{\frac{8}{7} \times 65} = \frac{7 \times 11 \times 15}{8 \times 65} = \frac{105 \times 110}{65 \times 80}.$$

To cut 3 threads per 2 in. Leadscrew 6 mm. pitch. No. of threads per inch $\frac{3}{2}$. No. of threads in the leadscrew per cm. $\frac{10}{6}$.

Solution: $$\frac{\frac{10}{6} \times 11 \times 15}{\frac{3}{2} \times 65} = \frac{10 \times 11 \times 5}{3 \times 65} = \frac{55 \times 100}{30 \times 65}.$$

To cut 36 threads per 7 in. Leadscrew, 7 mm. pitch. No. of threads per inch $\frac{36}{7}$. No. of threads in the leadscrew per cm. $\frac{10}{7}$.

Solution: $$\frac{\frac{10}{7} \times 11 \times 15}{\frac{36}{7} \times 65} = \frac{10 \times 11 \times 15}{36 \times 65}$$

$$= \frac{5 \times 11}{6 \times 13} = \frac{50 \times 55}{60 \times 65}.$$

To cut 9·5 thread per 8 inch. Leadscrew, 10 mm. pitch. No. of threads per inch, $\frac{9 \cdot 5}{8}$.

Solution: $$\frac{11 \times 15}{\frac{9 \cdot 5}{8} \times 65} = \frac{8 \times 11 \times 15}{9 \cdot 5 \times 65} = \frac{110 \times 120}{65 \times 95}.$$

To cut 25 threads per $3\frac{3}{4}$ in. Leadscrew, 5 mm. pitch. No. of threads per inch, $\frac{25}{3\frac{3}{4}} = \frac{100}{15}$. No. of threads in the leadscrew per cm. = 2.

Solution: $$\frac{2 \times 11 \times 15}{\frac{100}{15} \times 65} = \frac{2 \times 11 \times 15 \times 15}{100 \times 65}$$

$$= \frac{55 \times 90}{65 \times 100}.$$

To cut a $2\frac{1}{2}$ in. pitch. Leadscrew, 10 mm. pitch. No. of threads per inch, $\frac{1}{2\frac{1}{2}} = \frac{2}{5}$.

Solution : $\frac{11 \times 15}{\frac{2}{5} \times 65} = \frac{5 \times 11 \times 15}{2 \times 65} = \frac{110 \times 75}{20 \times 65}$.

To cut 2 threads per $6\frac{1}{2}$ in. Leadscrew, 25 mm. pitch. No. of threads per inch, $\frac{2}{6\frac{1}{2}} = \frac{4}{13}$. No. of threads in the leadscrew per cm., $\frac{1}{2\frac{1}{2}} = \frac{2}{5}$.

Solution : $\frac{\frac{2}{5} \times 11 \times 15}{\frac{4}{13} \times 65} = \frac{2 \times 13 \times 11 \times 15}{4 \times 5 \times 65} = \frac{55 \times 60}{40 \times 25}$.

(*f*) *The Cutting of Metric Threads on a Lathe with English Leadscrew.*

To some extent the fourth case resembles the third. The proportion 10 : 25·4 also holds good, though with an opposite meaning.

Use is also made in this instance of the fact that 6·5 in. is equivalent to 16·5 cm.

Suppose, for example, that the leadscrew has a 1 inch pitch and 10 threads per cm. have to be cut, i.e. a 1 mm. pitch, then, when the leadscrew has completed 6·5 revolutions, the lathe spindle should have made 165 revolutions, which can be formulated

$$\frac{\text{No. of threads in the leadscrew per } 6{\cdot}5 \text{ in.}}{\text{No. of threads to be cut per } 165 \text{ mm.}} = \frac{6{\cdot}5}{165}.$$

The numerator of the fraction will thus, for a given lathe, always be equivalent to the number of threads per inch in the leadscrew × the factor 6·5; the denominator being equivalent to a fraction, the numerator of which is the factor 165, and the length in mm. of the thread to be cut, the denominator.

For example, a 2 mm. pitch is to be cut on a lathe having a leadscrew of 2 threads per inch, then
the numerator will be $2 \times 6{\cdot}5 = 13$
and the denominator will be $\frac{165}{2}$.

For this particular lathe the numerator will always be 13.

The first resolvent of the fraction is a whole number obtained from the denominator by placing the denominator of the fraction, which is the denominator of the compound fraction in the numerator, thus $\frac{2 \times 13}{165}$.

No useful purpose, however, is effected by this alteration every time. The pitch of the thread to be cut is accordingly placed directly in the numerator, the fraction then being definitely formulated as follows :—

$$\text{Numerator} = \text{Pitch in mm. of thread to be cut} \times \text{No. of threads in the leadscrew per inch} \times 6{\cdot}5$$

$$\text{Denominator} = 165.$$

Attention must here be directed to the fact that whenever the length of the thread to be cut is a fraction, it must never be resolved into a decimal, but must always be placed in the numerator as a vulgar fraction, so that compound fractions may be resolvable from numerator and denominator by multiplication of both.

The following examples, from the simplest to the most complicated, will make clear what has been stated above :—

To cut a screw of 5 threads per cm. Leadscrew 2 threads per inch.

To be cut a 2 mm. pitch.

Solution : $\frac{2 \times 2 \times 6{\cdot}5}{165} = \frac{2 \times 13}{11 \times 15} = \frac{20 \times 65}{75 \times 110}$.

To cut a 3·5 mm. pitch. Leadscrew 2 threads per inch.

Solution : $\frac{3{\cdot}5 \times 13}{11 \times 15} = \frac{35 \times 65}{75 \times 110}$.

To cut a screw of 3 threads per cm. Leadscrew 2 threads per inch.

To be cut a $\frac{10}{3}$ mm. pitch.

Solution: $\frac{\frac{10}{3} \times 13}{11 \times 15} = \frac{10 \times 13}{3 \times 11 \times 15} = \frac{10 \times 13}{11 \times 45} = \frac{20 \times 65}{45 \times 110}$.

To cut a screw of 8 threads per 11 mm. Leadscrew 2 threads per inch.

To be cut a $\frac{11}{8}$ mm. pitch.

Solution: $\frac{\frac{11}{8} \times 13}{11 \times 15} = \frac{13}{8 \times 15} = \frac{20 \times 65}{100 \times 120}$.

To cut a screw of 5 threads per 18 mm. Leadscrew 2 threads per inch.

Solution: $\frac{\frac{18}{5} \times 13}{11 \times 15} = \frac{13 \times 18}{5 \times 11 \times 15} = \frac{6 \times 13}{11 \times 25} = \frac{30 \times 65}{55 \times 125}$.

To cut a screw of 4 threads per 7 mm. Leadscrew $2\frac{1}{2}$ threads per inch.

Solution: $\frac{\frac{7}{4} \times 2\frac{1}{2} \times 6\frac{1}{2}}{11 \times 15} = \frac{7 \times 13}{4 \times 2 \times 6 \times 11} = \frac{35 \times 65}{110 \times 120}$.

To cut a $7\frac{1}{2}$ mm. pitch. Leadscrew $2\frac{1}{2}$ threads per inch.

Solution: $\frac{7\frac{1}{2} \times 2\frac{1}{2} \times 6\frac{1}{2}}{11 \times 15} = \frac{5 \times 13}{11 \times 8} = \frac{50 \times 65}{55 \times 80}$.

To cut a $10\frac{1}{2}$ mm. pitch. Leadscrew 1 thread per inch.

Solution: $\frac{10\frac{1}{2} \times 6\frac{1}{2}}{11 \times 15} = \frac{21 \times 13}{4 \times 11 \times 15} = \frac{7 \times 13}{11 \times 20} = \frac{35 \times 65}{55 \times 100}$.

To cut a 42 mm. pitch. Leadscrew 1 inch pitch.

Solution: $\frac{42 \times 6 \cdot 5}{11 \times 15} = \frac{42 \times 13}{2 \times 11 \times 15} = \frac{7 \times 13}{5 \times 11} = \frac{70 \times 65}{50 \times 55}$.

To cut a screw of 13 threads per 5 mm. Leadscrew 4 threads per inch.

Solution: $\frac{\frac{5}{13} \times 4 \times 6\frac{1}{2}}{11 \times 15} = \frac{2 \times 5}{7 \cdot 5 \times 22} = \frac{20 \times 25}{75 \times 110}$.

(*g*) *The Wheel with 127 Teeth.*

In addition to the equivalent 6·5 in. = 16·5 cm., which has been employed in the third and fourth cases, there is still another way of cutting English thread on a lathe with metric leadscrew, or *vice versâ*, which is, by making use of a wheel with 127 teeth.

The proportion between the cm. and the inch of 10 : 25·4 can be resolved into one of 50 : 127.

127 is not divisible further, and so, if a wheel with 127 teeth be employed, this factor can be placed either in the numerator or the denominator.

The third and fourth cases will then resemble the first, seeing that it is now possible to express the English thread in mm., whether it be the threads in the leadscrew or the threads in the screw to be cut. The fraction will thus be—

$$\begin{array}{l}\text{Numerator} = \text{Pitch to be cut in mm.} \\ \text{Denominator} = \text{Pitch of leadscrew in mm.}\end{array} \quad \text{or}$$

$$\begin{array}{l}\text{Numerator} = \text{No. of threads in leadscrew per inch} \\ \text{Denominator} = \text{No. of threads to be cut per cm.} \times 2{\cdot}54\end{array}$$

The following examples will clearly indicate what is meant :—

To cut a 2 mm. pitch. Leadscrew 2 threads per inch. Leadscrew pitch 12·7 mm. :—

$$\begin{array}{l}\text{Numerator} = 2 \\ \text{Denominator} = 12{\cdot}7\end{array} = \frac{20}{127}$$

The foregoing example, when worked out as per the last comparison, will yield the same result, seeing that :—

2 mm. = 5 threads per cm.

$$\begin{array}{l}\text{Numerator} = 2 \\ \text{Denominator} = 5 \times 2{\cdot}54\end{array} = \frac{2}{12{\cdot}7} = \frac{20}{127}$$

To cut 3 threads per cm. Leadscrew 2 threads per inch.

Solution : $\frac{2}{3 \times 2{\cdot}54} = \frac{2}{6 \times 1{\cdot}27} = \frac{4 \times 50}{6 \times 127} = \frac{40 \times 50}{60 \times 127}$

or, according to *first comparison*,

1 pitch = $\frac{10}{3}$ mm.

Numerator $= \frac{\frac{10}{3}}{12{\cdot}7} = \frac{10}{3 \times 12{\cdot}7} = \frac{40 \times 50}{60 \times 127}$.
Denominator $= 12{\cdot}7$

To cut 7 threads per 44 mm. Leadscrew 2 threads per inch.

Solution : $\frac{\frac{44}{7}}{12{\cdot}7} = \frac{44}{7 \times 12{\cdot}7} = \frac{40 \times 55}{35 \times 127}$

To cut a 9 mm. pitch. Leadscrew $2\frac{1}{2}$ threads per inch.

Solution : $\frac{9}{\frac{25{\cdot}4}{2{\cdot}5}} = \frac{9 \times 25}{254} = \frac{45 \times 125}{50 \times 127}$.

To cut 28 threads per 45 mm. Leadscrew 4 threads per inch.

Solution : $\frac{\frac{45}{28}}{\frac{25{\cdot}4}{4}} = \frac{45 \times 4}{28 \times 25{\cdot}4} = \frac{45 \times 50}{70 \times 127}$

To cut 1 in. Whitworth-thread = 8 threads per inch. Leadscrew 10 mm. Pitch to be cut = $\frac{25{\cdot}4}{8}$ mm.

Solution : $\frac{\frac{25{\cdot}4}{8}}{10} = \frac{25{\cdot}4}{8 \times 10} = \frac{20 \times 127}{10 \times 100}$.

When cutting metric thread on a lathe with English leadscrew, the wheel with 127 teeth is always to be found amongst the wheels driven, whilst, when cutting English thread on a lathe with metric leadscrew, it is found among the drivers.

To cut 3 in. Whitworth-thread = $3\frac{1}{2}$ threads per inch. Leadscrew 10 mm. pitch.

Solution : $\frac{25{\cdot}4}{3{\cdot}5 \times 10} = \frac{20 \times 127}{35 \times 100}$.

To cut 4 in. gas thread = 11 threads per inch. Leadscrew 10 mm. pitch.

Solution : $\frac{25 \cdot 4}{11 \times 10} = \frac{20 \times 127}{100 \times 110}$.

To cut 3 threads per $8\frac{1}{2}$ in. $= \frac{3}{8\frac{1}{2}}$ inch pitch. Leadscrew 10 mm. pitch.

Solution : $\frac{\frac{25 \cdot 4}{3}}{\frac{8 \cdot 5}{10}} = \frac{25 \cdot 4 \times 8 \cdot 5}{3 \times 10} = \frac{85 \times 127}{25 \times 60}$.

To cut 9 threads per 11 in. Leadscrew 25 mm. pitch.

Solution : $\frac{\frac{25 \cdot 4}{11}}{\frac{9}{25}} = \frac{9 \times 25 \cdot 4}{11 \times 25} = \frac{45 \times 127}{55 \times 125}$.

To cut 7 threads per 3 in. Leadscrew 7 mm. pitch.

Solution : $\frac{3 \times 25 \cdot 4}{7 \times 7} = \frac{30 \times 127}{35 \times 70}$.

To cut 24 threads per 9 in. Leadscrew 5 mm. pitch.

Solution : $\frac{9 \times 25 \cdot 4}{24 \times 5} = \frac{45 \times 127}{50 \times 60}$.

(*h*) *Method for Calculating Approximate Fractions.*

Before commencing with the actual calculation, the question was propounded under heading (*b*) on page 15 : " What change-wheels are to be found on a lathe ? " This was indeed imperative, as the change-wheels actually present on the lathe have invariably to be taken into account, first of all because the fraction must be resolved into numbers corresponding to the change-wheels, and then, because the same factors which go to make up the fraction must also be found in the change-wheels. Should the fraction contain a factor not to be met with in the change-wheels, then, according to the methods now in vogue, a suitable set of wheels could not be found,

consequently, the thread in question could not be cut without obtaining one or more wheels making up the requisite factors, which, of course, would not be possible, as a certain thread is generally required to be cut without notice, and there is, therefore, no chance of either making or obtaining suitable wheels.

Will such cases often occur? Not as a rule. The examples already given clearly show that even in the case of threads which vary very considerably, the wheels necessary for cutting a true thread can be found.

In the set of change-wheels, given on page 15, the following factors were found: 2, 3, 5, 7, 11, 13, 17, 19, 23; the factor 23 was not met with in the second set, whilst on many lathes the factors 17, 19, and 23 are absent.

If factors appear in the fraction composed of the thread to be cut and the leadscrew, which cannot be found in the change-wheels, then such a thread cannot be cut accurately.

If it is absolutely necessary to cut such a thread, a fraction must be sought for which approaches the correct fraction as nearly as possible.

Lack of knowledge of the correct method of finding out a fraction approximating the true fraction as closely as possible, too often results in the calculation being skipped over, and a fraction being chosen which actually gives a thread differing considerably from the one required.

In addition, the fact is too often lost sight of that an approximate fraction will still result in an *unserviceable* thread.

Suppose, for example, a fraction is found which yields a thread differing only 0·05 mm. from the thread of the nut to fit which the thread has to be cut. At first sight the difference appears trifling, but the error which has been made is really very *great*, so great, indeed, that the thread obtained is wholly *useless*. It must of course not be forgotten that each thread increases the error, which at the end of 20 threads will result in a difference of $20 \times 0{\cdot}05$ mm. = 1 mm. Suppose, further, that a thread has to be cut of 23 threads per inch, the pitch being $\frac{23}{25{\cdot}4} = 1\frac{20}{254}$ mm. With a difference of

0·05 mm. per thread, the difference at the end of 10 threads will be equivalent to one-half of the thread, whilst at the end of 23 threads, the difference will amount to the entire thread.

The foregoing example clearly demonstrates that only fractions differing by some thousandths of a millimetre, or some ten thousandths of an inch, can be employed.

How can such an approximate fraction be arrived at?

Regular practice often enables one to find a fraction which approaches very closely, without the assistance of any method.

In one of his note-books the writer found a fraction which had been discovered, apart from any method, for the cutting of a 3·7 mm. thread on a lathe with a leadscrew having a pitch of 10 mm.

For this thread there were no change-wheels, for a wheel in which the factor 37 appears, which is indivisible, is not to be found among an ordinary set of change-wheels.

For this reason, according to the notes in question, the fraction $\frac{77}{208}$ was chosen, for which change-wheels could be found, since $\frac{77}{208} = \frac{7 \times 11}{13 \times 16} = \frac{35 \times 55}{65 \times 80}$.

Seeing that the difference between $\frac{3{\cdot}7}{10}$ and $\frac{77}{208}$ is simply the difference between 3·7 and 3·701 = 0·001 mm., so that after 10 threads the difference is still only 0·01 mm., which may be considered near enough for all practical purposes.

Such groping about in the dark, however, is not at all methodical, can take a very long time, and, finally, may not lead to any actual result.

The compound fraction, however, supplies us with a ready means of discovering a fraction which approximates sufficiently to permit the obtaining of what is practically an accurate thread.

Suppose the fraction to consist of two numbers, the numerator and denominator of which are both positive.

Let these numbers be represented by A and B, and A > B. This can then be represented

$$\frac{A}{B} = a_1 + \frac{r_1}{B} \quad r_1 < B \quad \text{or} \quad B > r_1.$$

Taking the reverse of the last-named fraction, the reduction can then be further continued,

$$\frac{B}{r_1} = a_2 + \frac{r_2}{r_1} \quad r_2 < r_1 \quad \text{or} \quad r_1 > r_2.$$

Continuing further

$$\frac{r_1}{r_2} = a_3 + \frac{r_3}{r_2} \quad r_3 < r_2 \quad \text{or} \quad r_2 > r_3$$

which can be continued ad infinitum, and can thus be expressed

$$\frac{r_{n-2}}{r_{n-1}} = a_n + \frac{r_n}{r_{n-1}},$$

in which

$$r_n < r_{n-1} \text{ or } r_{n-1} > r_n.$$

The quotients $a_1, a_2, a_3 \ldots a_n$, are termed *indicators*. By substitution can be obtained

$$\frac{A}{B} = a_1 + \frac{1}{a_2 + \frac{r_2}{r_1}}$$

$$\frac{A}{B} = a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \frac{r_3}{r_2}}}, \text{ etc., etc.}$$

or,

$$\frac{A}{B} = a_1 + \cfrac{1}{a_2 \cfrac{1}{a_3 \cfrac{1}{a_4 \cfrac{1}{a_5 \cfrac{1}{a_6 \cfrac{1}{a_7 \cfrac{1}{a_8 + \cfrac{1}{a_n + \frac{r_n}{r_{n-1}}}}}}}}}}$$

If $\frac{r_n}{r_{n-1}} = 0$, then the number of terms is finite, in which case the fraction $\frac{A}{B}$ is determinable, in that it can finally be divided without leaving a remainder.

If the proportion $\frac{A}{B}$ be indeterminable, and cannot consequently be expressed by a fraction with exactness, then there will be no end to the divisions, in which case the number of terms of the compound fraction will be *infinite*.

Every *indeterminable* number may be regarded as the *limit* of an indefinite, non-recurring fraction. The *limit* of a repeating decimal fraction is a determinable proportion, e.g. the limit of $0\cdot\dot{3}$ is $\frac{1}{3}$.

To apply the foregoing to a definite fraction.

(1) Given A > B, for example.

To express the fraction $\frac{9976}{6961}$ as a compound fraction.

$$\frac{9976}{6961} = 1 + \cfrac{3015}{6961} + \cfrac{1}{2 + \cfrac{931}{3015} + \cfrac{1}{3 + \cfrac{222}{931} + \cfrac{1}{4 + \cfrac{43}{222} + \cfrac{1}{5 + \cfrac{7}{43} + \cfrac{1}{6 + \frac{1}{7}}}}}}$$

The *indicators* are thus 1, 2, 3, 4, 5, 6, 7.

Consequently $\frac{9976}{6961}$ as a compound fraction =

$$1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{6 + \frac{1}{7}}}}}}$$

(2) Given A < B, for example.

To express $\frac{113}{355}$ as a compound fraction.

$$\frac{113}{355} = \frac{1}{\frac{355}{113}} = \cfrac{1}{3 + \cfrac{16}{113} + \cfrac{1}{7 + \frac{1}{16}}}$$

If $\frac{A}{B} < 1$, the first indicator can then be expressed by 0, in which case the indicators will be

$$0,\ \tfrac{1}{3},\ \tfrac{1}{7} \text{ and } \tfrac{1}{16};$$

thus

$$\frac{113}{355} = 0 + \cfrac{1}{3 + \cfrac{1}{7 + \frac{1}{16}}} \text{ as compound fraction.}$$

(3) Express the compound fraction 4 as an ordinary fraction.

3
2
1
4

$$\frac{A}{B} = 4 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{4}}}} \quad \begin{matrix} = 4\frac{1}{4} \\ = 3\frac{5}{14} \\ = 2\frac{4}{5} \end{matrix}$$

So that $\frac{A}{B} = 4\frac{14}{47} = \frac{202}{47}$.

(4) Express the compound fraction $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{1}\ \frac{1}{5}$ as an ordinary fraction.

$$\frac{A}{B} = \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{1 + \frac{1}{5}}}}}$$

$$\left.\begin{aligned} 1 + \tfrac{1}{5} &= \tfrac{6}{5} \\ 4 + \tfrac{1}{\frac{6}{5}} &= 4 + \tfrac{5}{6} = \tfrac{29}{6} \\ 3 + \tfrac{1}{\frac{29}{6}} &= 3 + \tfrac{6}{29} = \tfrac{93}{29} \\ 2 + \tfrac{1}{\frac{93}{29}} &= 2 + \tfrac{29}{93} = \tfrac{215}{93} \end{aligned}\right\} = \frac{1}{\frac{215}{93}}$$

$\frac{A}{B}$ is thus $= \frac{93}{215}$.

The general formula can now be expressed by putting letters in place of the figures given in the foregoing examples.

Given the compound fraction $\begin{matrix} a \\ b \\ c \\ d \end{matrix}$, determine the ordinary fraction.

$$\frac{A}{B} = a + \frac{1}{b + \cfrac{1}{c + \frac{1}{d}}} = a + \frac{cd+1}{bcd+b+d} = \frac{abcd+ab+ad+cd+1}{bcd+b+d}$$

$$b + \frac{1}{c + \frac{1}{d}} = b + \frac{d}{cd+1} = \frac{bcd+b+d}{cd+1}$$

$$c + \frac{1}{d} = \frac{cd+1}{d}$$

given that

$$\left.\begin{aligned} a &= 1 \\ b &= 2 \\ c &= 3 \\ d &= 4 \end{aligned}\right\} \text{ then } \begin{aligned} abcd &= 24 \\ ab &= 2 \\ ad &= 4 \\ cd &= 12 \\ \hline 42 + 1 &= 43 = \text{the numerator.} \end{aligned}$$

$$\begin{aligned} bcd &= 24 \\ b &= 2 \\ d &= 4 \\ \hline 30 &= \text{the denominator.} \end{aligned}$$

so that in this case the value of the fraction $\frac{A}{B} = \frac{43}{30}$.

For any given value of a, b, c, and d, the fraction can be immediately determined from the fraction

$$\frac{abcd + ab + ad + cd + 1}{bcd + b + d}.$$

To take the reverse. Given the ordinary fraction

$$\frac{(ab + 1)c + a}{bc + 1},$$

determine the compound fraction.

$$\frac{(ab + 1)c + a}{bc + 1} = \frac{abc + c + a}{bc + 1} = a + \frac{c}{bc + 1}$$

$$= a + \frac{1}{b + \frac{1}{c}} = \text{indicators.}$$

The indicators are thus a, b, and c.

Given that in the foregoing fraction the indicators have the following value: $a = 2$, $b = 3$, $c = 7$.

Then reversing the order of things in the foregoing example

$$\frac{(ab + 1)c + a}{bc + 1} = \frac{(2 \times 3 + 1)4 + 2}{3 \times 4 + 1} = \frac{28 + 2}{12 + 1} = \frac{30}{13}.$$

The indicators for the fraction $\frac{30}{13}$ are thus 2, 3, and 4.

The foregoing consequently proves :—

(1) That every *determinable* fraction may be expressed as a *finite* compound fraction.

(2) That every *finite* compound fraction may be expressed as a *determinable* fraction.

Compound fractions may be divided into :—

(*a*) Symmetric.

(*b*) Periodic { wholly. / partially.

If terms and compound fraction be expressed as

$$\frac{A}{B} = \underbrace{(a_1, a_2, a_3, a_4, \ldots a_n)}_{\text{indicators}}$$

then

$$\frac{A}{B} = (a_1, a_2, a_3, \ldots a_3, a_2, a_1)$$

is termed a symmetric compound fraction because the indicators end in the same order of sequence as they began ; and

$$\frac{A}{B} = (a_1, a_2, a_3, a_4, a_2, a_3, a_4, a_2, a_3, a_4 \ldots)$$

is termed a periodic compound fraction, because the indicators a_2, a_3, a_4 occur periodically. In both cases the number of terms is infinite.

The Finding-out of Approximating Fractions.

Whenever the factors of a fraction, according to which a thread is required to be cut, are not represented by the change-wheels belonging to the lathe, it is impossible, as has already been demonstrated above, to cut a theoretically accurate thread, but an attempt can be made to discover a fraction, the value of which approaches that of the real fraction so closely that the two may be regarded as practically identical.

Such an approximating fraction can be found by resolving the fraction into a compound fraction, and terminating this at the second, third, fourth, fifth, etc., indicator.

For example—

$$\frac{A}{B} = a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}$$

For the first quotient substitute

$$a_1 = \frac{P_1}{Q_1} = \frac{a_1}{1},$$

then the second quotient will be

$$\frac{P_2}{Q_2} = a_1 + \frac{1}{a_2} = \frac{a_1 a_2 + 1}{a_2}$$

the third quotient being

$$\frac{P_3}{Q_3} = a_1 + \frac{1}{a_2 + \frac{1}{a_3}} = \frac{a_3 (a_1 a_2 + 1) + a_1}{a_3 a_2 + 1}, \text{ etc. etc.}$$

$\frac{P_1}{Q_1}, \frac{P_2}{Q_2}, \frac{P_3}{Q_3}$ are the reduced approximated fractions, the values of which are alternately greater and smaller than the value of $\frac{A}{B}$, and they approach more and more closely to $\frac{A}{B}$, which may consequently be regarded as their *limit.*

The greater the number of indicators, the smaller the difference between the *approximating* fraction and the *exact* value of $\frac{A}{B}$.

The following connection can be established between the *approximating* fractions and the indicators :—

$P_1 = a_1$	$P_2 = a_1 a_2 + 1$	$P_3 = a_3 (a_1 a_2 + 1) + a_1$
$Q_1 = 1$	$Q_2 = a_2$	$Q_3 = a_3 a_2 + 1$

consequently

$$\frac{P_3}{Q_3} = \frac{a_3 (a_1 a_2 + 1) + a_1}{a_3 a_2 + 1} = \frac{a_3 P_2 + P_1}{a_3 Q_2 + Q_1},$$

consequently

$$P_3 = a_3 P_2 + P_1 \quad \text{and} \quad Q_3 = a_3 Q_2 + Q_1.$$

It follows, therefore, as a general rule that

$$P_n = a_n P_{n-1} + P_{n-2} \quad \text{and} \quad Q_n = a_n Q_{n-1} + Q_{n-2},$$

and this can be applied in the following manner:—

(1) Given the fraction $\frac{51}{16}$. Determine the compound fraction, i.e. the indicators, and find an approximative fraction.

$$\frac{A}{B} = \frac{51}{16} = 3 + \frac{1}{5 + \frac{1}{3}} \qquad \begin{aligned} a_1 &= 3 \\ a_2 &= 5 \\ a_3 &= 3 \end{aligned}$$

$$\frac{A}{B} = \frac{P_1}{Q_1} = \frac{a_1}{1} = 3, \text{ limit approached}$$

$$\frac{A}{B} = \frac{P_2}{Q_2} = \frac{3 \times 5 + 1}{5} = \frac{16}{5}, \text{ limit approached still closer}$$

$$\frac{A}{B} = \frac{P_3}{Q_3} = \frac{3(16) + 3}{15 + 1} = \frac{51}{16}, \text{ the exact value.}$$

(2) Given the fraction $\frac{3370}{399}$.

$$\frac{A}{B} = \frac{3370}{399} = 8 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{7 + \frac{1}{6}}}}$$

$a_1 = 8$ $\quad P_1 = a_1 = 8 \quad Q_1 = 1 \quad \frac{P_1}{Q_1} = 8$

$a_2 = 2$ $\quad P_2 = a_1 a_2 + 1 = 17 \quad Q_2 = a_2 = 2 \quad \frac{P_2}{Q_2} = \frac{17}{2}$

$a_3 = 4$ $\quad P_3 = a_3(a_1 a_2 + 1) + a_1 = 76 \quad Q_3 = a_2 + 1 = 9 \quad \frac{P_3}{Q_3} = \frac{76}{9}$

$a_4 = 7$ $\quad P_4 = a_4(a_3(a_1 a_2 + 1) + a_1 a_2 + 1) = 549 \quad Q_4 = a_4(a_3 a_2 + 1) + a_2 = 65 \quad \frac{P_4}{Q_4} = \frac{549}{65}$

$a_5 = 6$ $\quad P_5 = a_5 \underbrace{(a_4(a_3(a_1 a_2 + 1) + a_1 a_2 + 1))}_{6 \times 549} + \underbrace{a_3(a_1 a_2 + 1) + a_1}_{76} = 3370$

$\quad Q_5 = a_5 \underbrace{(a_4(a_3 a_2 + 1) + a_2)}_{6 \times 65} + \underbrace{a_3 a_2 + 1}_{9} = 399$ $\quad \frac{P_5}{Q_5} = \frac{3370}{399}$

The approximating fractions are thus

$$\frac{8}{1}, \frac{17}{2}, \frac{76}{9}, \frac{549}{65}, \frac{3370}{399}.$$

(3) Determine the compound fraction and the approximating fractions of the number 2·718281828459.

$$\frac{A}{B} = \frac{2718281828459}{10^{12}} = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4}}}}}$$

$a_1 = 2$	$P_1 = 2$	$Q_1 = 1$	$\frac{P_1}{Q_1} = \frac{2}{1}$
$a_2 = 1$	$P_2 = 2 \times 1 + 1 = 3$	$Q_2 = 1$	$\frac{P_2}{Q_2} = \frac{3}{1}$
$a_3 = 2$	$P_3 = 2 \times 3 + 2 = 8$	$Q_3 = 2 \times 1 + 1 = 3$	$\frac{P_3}{Q_3} = \frac{8}{3}$
$a_4 = 1$	$P_4 = 1 \times 8 + 3 = 11$	$Q_4 = 1 \times 3 + 1 = 4$	$\frac{P_4}{Q_4} = \frac{11}{4}$
$a_5 = 1$	$P_5 = 1 \times 11 + 8 = 19$	$Q_5 = 1 \times 4 + 3 = 7$	$\frac{P_5}{Q_5} = \frac{19}{7}$
$a_6 = 4$	$P_6 = 4 \times 19 + 11 = 87$	$Q_6 = 4 \times 7 + 4 = 32$	$\frac{P_6}{Q_6} = \frac{87}{32}$

(4) Determine the approximating fractions for the number $\pi = 3{\cdot}14159265359\ldots$.

$$\frac{A}{B} = \frac{314159265359}{10^{11}} = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6}}}}}}}$$

$a_1 = 3$	$P_1 = 3$	$Q_1 = 1$	etc.
$a_2 = 7$	$P_2 = 22$	$Q_2 = 7$	
$a_3 = 15$	$P_3 = 333$	$Q_3 = 106$	
$a_4 = 1$	$P_4 = 355$	$Q_4 = 113$	
$a_5 = 292$	$P_5 = 103993$	$Q_5 = 33102$	
$a_6 = 1$			
$a_7 = 1$		etc. etc.	
$a_8 = 6$			

The approximating fractions are, consequently,

$$\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102} \quad \text{etc. etc.}$$

From which the following can be determined :—

Axiom 1.—The difference between two successive approximating fractions is, the signs not being taken into consideration, equal to the unit divided by the product of its numerators ; or, in general,

$$V_n = \frac{P_n}{Q_n} - \frac{P_{n+1}}{Q_{n+1}} = \frac{(-1)^n}{Q_n Q_{n+1}}.$$

Should there also be three successive approximating fractions,

$$\frac{P_{n-1}}{Q_{n-1}}, \frac{P_n}{Q_n}, \frac{P_{n+1}}{Q_{n+1}},$$

the first will then be greater than the second, the second being smaller than the third, etc.

Example (see page 42) :

$$\frac{A}{B} = \frac{3370}{399};$$

the approximating fractions are

$$\underbrace{\frac{8}{1}, \frac{17}{2}, \frac{76}{9}, \frac{549}{65}}$$

$$V_n = \frac{-1}{2}, \frac{+1}{18}, \frac{-1}{585}$$

Axiom 2.—The difference between the exact value of the fraction $\frac{A}{B}$ and one of the approximating fractions will invariably be less than the unit divided by the product of the denominators of this approximating fraction and those following, and also less than the unit divided by the square of the

denominator of the fraction under consideration; or, in general:

$$\frac{A}{B} = \frac{P_n}{Q_n} < \frac{(-1)^n}{Q_n Q_{n+1}}$$

$$\frac{A}{B} - \frac{P_n}{Q_n} < \frac{(-1)^n}{Q_n^2}$$

$$\frac{(-1)^n}{Q_n Q_{n+1}} < \frac{(-1)^n}{Q_n^2}$$

$$\frac{A}{B} - \frac{P_n}{Q_n} < \frac{1}{Q_n (Q_n + Q_{n-1})}$$

Application: $\frac{A}{B} = \frac{3370}{399}$

$$\frac{P_1}{Q_1} = \frac{8}{1} \qquad \frac{P_2}{Q_2} = \frac{17}{2} \qquad \frac{P_3}{Q_3} = \frac{76}{9} \qquad \frac{P_4}{Q_4} = \frac{549}{65}$$

$$\frac{3370}{399} - 8 < \frac{-1}{2} = \frac{178}{399} < -\frac{1}{2}$$

$$\frac{3370}{399} - \frac{17}{2} < \frac{1}{18}$$

$$\frac{3370}{399} - \frac{17}{2} < \frac{1}{4}$$

$$\frac{1}{18} < \frac{1}{4}$$

$$\frac{3370}{399} - \frac{17}{2} < \frac{1}{16} \quad \text{etc. etc.}$$

From which it follows that in order to obtain an approximating fraction, differing only a millionth part from the exact value, the denominator must consist of at least 4 figures.

The differences between two successive approximating fractions become continually smaller, and are alternately positive and negative. The difference approaches *nil*, and consequently the limit of the approximating fraction to the exact value of $\frac{A}{B}$.

By interpolation another fraction can still be found between two approximating fractions.

General term :—

$$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q_{n-1} + Q_{n-2}}.$$

By taking in place of a_n the values 1, 2, 3 . . . . . (a_{n-1}), other fractions can be interpolated between $\frac{P_{n-2}}{Q_{n-2}}$ and $\frac{P_n}{Q_n}$, both of which form an increasing or diminishing chain, as they both have the same sign.

(1) Required, the interpolated fractions between $\frac{P_2}{Q_2} = \frac{17}{2}$ and $\frac{P_4}{Q_4} = \frac{549}{65}$ of the fraction $\frac{A}{B} = \frac{3370}{399}$ (page 42).

$a_4 = 7$ $\quad a_{n-1} = 6,\ 5,\ 4,\ 3,\ 2$ and 1.

$$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q_{n-1} + Q_{n-2}}$$

$P_n = 549$ $\quad Q_n = 65$

$P_{n-1} = 76$ $\quad Q_{n-1} = 9$

$P_{n-2} = 17$ $\quad Q_{n-2} = 2$

$$\frac{P_n}{Q_n} = \frac{6 \times 76 + 17}{6 \times 9 + 2} = \frac{473}{56} \qquad \frac{P_n}{Q_n} = \frac{3 \times 76 + 17}{3 \times 9 + 2} = \frac{245}{29}$$

$$\frac{P_n}{Q_n} = \frac{5 \times 76 + 17}{5 \times 9 + 2} = \frac{397}{47} \qquad \frac{P_n}{Q_n} = \frac{2 \times 76 + 17}{2 \times 9 + 2} = \frac{169}{20}$$

$$\frac{P_n}{Q_n} = \frac{4 \times 76 + 17}{4 \times 9 + 2} = \frac{321}{38} \qquad \frac{P_n}{Q_n} = \frac{1 \times 76 + 17}{1 \times 9 + 2} = \frac{93}{11}$$

The fractions $\frac{93}{11}, \frac{169}{20}, \frac{245}{29}, \frac{321}{38}, \frac{397}{47}, \frac{473}{56}$, lie thus between the fractions $\frac{17}{2}$ and $\frac{549}{65}$, which are approximating fractions of $\frac{3370}{399}$.

(2) Required, the interpolated fractions between $\frac{P_1}{Q_1} = \frac{8}{1}$ and $\frac{P_3}{Q_3} = \frac{76}{9}$ of the same fraction.

$a_3 = 4$ $\quad a_{n-1} = 3,\ 2,$ and 1

$$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q_{n-1} + Q_{n-2}}$$

$P_n = 76 \quad Q_n = 9$

$P_{n-1} = 17 \quad Q_{n-1} = 2$

$P_{n-2} = 8 \quad Q_{n-2} = 1$

$$\frac{P_n}{Q_n} = \frac{3 \times 17 + 8}{3 \times 2 + 1} = \frac{59}{7}$$

$$\frac{P_n}{Q_n} = \frac{2 \times 17 + 8}{2 \times 2 + 1} = \frac{42}{5}$$

$$\frac{P_n}{Q_n} = \frac{1 \times 17 + 8}{1 \times 2 + 1} = \frac{25}{3}$$

consequently, the approximating fractions $\frac{59}{7}, \frac{42}{5}, \frac{25}{3}$, lie between $\frac{8}{1}$ and $\frac{76}{9}$.

(3) Required, the interpolated fractions $\frac{P_3}{Q_3} = \frac{76}{9}$ and $\frac{P_5}{Q_5} = \frac{3370}{399}$.

$a_5 = 6$ $\quad a_{n-1} = 5,\ 4,\ 3,\ 2$ and 1

$P_n = 3370 \quad Q_n = 399$

$P_{n-1} = 549 \quad Q_{n-1} = 65$

$P_{n-2} = 76 \quad Q_{n-2} = 9$

$$\frac{P_n}{Q_n} = \frac{5 \times 549 + 76}{5 \times 65 + 9} = \frac{2821}{334}$$

$$\frac{P_n}{Q_n} = \frac{4 \times 549 + 76}{4 \times 65 + 9} = \frac{2272}{269}$$

$$\frac{P_n}{Q_n} = \frac{3 \times 549 + 76}{3 \times 65 + 9} = \frac{1723}{204}$$

$$\frac{P_n}{Q_n} = \frac{2 \times 549 + 76}{2 \times 65 + 9} = \frac{1174}{139}$$

$$\frac{P_n}{Q_n} = \frac{1 \times 549 + 76}{1 \times 65 + 9} = \frac{625}{74}$$

the approximating fractions $\frac{625}{74}, \frac{1174}{139}, \frac{1723}{204}, \frac{2272}{269}$ and $\frac{2821}{334}$ thus lie between $\frac{76}{9}$ and $\frac{3370}{339}$.

Application.—Determine the compound fraction and the approximating fractions of the number 2·539954, so as to obtain another proportion as $\frac{25\cdot4}{100}$ or $\frac{12\cdot7}{50}$ for expressing the inch in cm.

$$\frac{A}{B} = \frac{2539954}{10^6} = 2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \frac{1}{11}}}}}}}}}}}}$$

The indicators are consequently :—

2,	1,	1,	5,	1,	3,	8,	2,	3,	2,	1,	1,	etc.
1	2	3	5	28	33	127	1049	2225				
1	1	1	2	11	13	50	413	876				

The following and the approximating fractions can be obtained by interpolation between fractions $\frac{28}{11}$ and $\frac{127}{50}$:—

$P_6 = 127$ $\quad Q_6 = 50$ $\quad a_6 = 3$

$P_5 = 33$ $\quad Q_5 = 13$ $\quad a_{6-1} = 2$ and 1

$P_4 = 28$ $\quad Q_4 = 11$

$$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q_{n-1} + Q_{n-2}} = \frac{2 \times 33 + 28}{2 \times 13 + 11} = \frac{94}{37}$$

$$\text{,,} \qquad = \frac{1 \times 33 + 28}{1 \times 13 + 11} = \frac{61}{24}$$

By interpolation between $\frac{P_5}{Q_5} = \frac{33}{13}$ and $\frac{P_7}{Q_7} = \frac{1049}{413}$, the following can be obtained :—

$$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q_{n-1} + Q_{n-2}} = \frac{1 \times 127 + 33}{1 \times 50 + 13} = \frac{160}{63}$$

$$\text{,,} = \frac{2 \times 127 + 33}{2 \times 50 + 13} = \frac{287}{113}$$

$$\text{,,} = \frac{3 \times 127 + 33}{3 \times 50 + 13} = \frac{414}{163}$$

$$\text{,,} = \frac{4 \times 127 + 33}{4 \times 50 + 13} = \frac{541}{213}$$

$$\text{,,} = \frac{5 \times 127 + 33}{5 \times 50 + 13} = \frac{668}{263}$$

$$\text{,,} = \frac{6 \times 127 + 33}{6 \times 50 + 13} = \frac{795}{313}$$

$$\text{,,} = \frac{7 \times 127 + 33}{7 \times 50 + 13} = \frac{922}{363}$$

so that the following approximating fractions can be found between $\frac{33}{13}$ and $\frac{1049}{413}$, viz.: $\frac{160}{63}$ $\frac{287}{113}$ $\frac{414}{163}$ $\frac{541}{213}$ $\frac{668}{263}$ $\frac{795}{313}$ and $\frac{922}{363}$.

A few Examples in Conclusion.

(1) It is required to cut $3\frac{1}{2}$ threads per $2\frac{11}{16}$ in. Leadscrew $\frac{1}{2}$ inch pitch.

Pitch to be cut $= \frac{2\frac{11}{16}}{3\frac{1}{2}}$. Leadscrew $\frac{1}{2}$ inch pitch.

Solution :—

$$\frac{2\frac{11}{16}}{3\frac{1}{2}} = \frac{2\frac{11}{16} \times 2}{3\frac{1}{2}} = \frac{43 \times 2 \times 2}{7 \times 16} = \frac{43}{7 \times 4} = \frac{43}{28}.$$

No wheel with 43 teeth is to be found, and the number 43 is indivisible. It will thus be necessary to find an approximating fraction.

$$\text{Compound fraction} = \frac{43}{28} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \frac{1}{2}}}}$$

Indicators :—1, 1, 1, 7, 2.

$a_1 = 1$	$P_1 = 1$	$Q_1 = 1$	$\frac{P_1}{Q_1} = 1$
$a_2 = 1$	$P_2 = 1 \times 1 + 1 = 2$	$Q_2 = 1$	$\frac{P_2}{Q_2} = 2$
$a_3 = 1$	$P_3 = 1 \times 2 + 1 = 3$	$Q_3 = 1 \times 1 + 1 = 2$	$\frac{P_3}{Q_3} = \frac{3}{2}$
$a_4 = 7$	$P_4 = 7 \times 3 + 2 = 23$	$Q_4 = 7 \times 2 + 1 = 15$	$\frac{P_4}{Q_4} = \frac{23}{15}$
$a_5 = 2$	$P_5 = 2 \times 23 + 3 = 49$	$Q_5 = 2 \times 15 + 2 = 32$	$\frac{P_5}{Q_5} = \frac{49}{32}$

Interpolating between $\frac{3}{2}$ and $\frac{49}{32}$.

$\frac{P_n}{Q_n} = \frac{a_n P_{n-1} + P_{n-2}}{a_n Q^{n-1} + Q^{n-2}} = \frac{1 \times 23 + 3}{1 \times 15 + 2} = \frac{26}{17}$ is obtained.

$\frac{43}{28} = 1{\cdot}5357.$

$\frac{49}{32} = 1{\cdot}5312$ which is 0·0045 less than the actual fraction.

$\frac{26}{17} = 1{\cdot}5294$ „ 0·0063 „ „ „

This difference occurs in every 2 threads, so that the actual difference per pitch is only 0 00225.

$\frac{49}{32}$ approaches most closely to these two, so that the wheels will consequently be

$$\frac{49}{32} = \frac{7 \times 7}{4 \times 8} = \frac{70 \times 70}{40 \times 80}.$$

(2) Required to cut a pitch of 3·7 mm. Leadscrew 10 mm.

Solution : $\frac{37}{100}$.

There being no wheel with 37 teeth, and the number 37 being indivisible, an approximating fraction will have to be found.

Compound fraction $= \frac{37}{100}$.

$$0 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{3}}}}}}$$

Indicators are thus 0, 2, 1, 2, 2, 1, 3.

$a_1 = 0$	$P_1 = 0$	$Q_1 = 1$	$\frac{P_1}{Q_1} = \frac{0}{1}$
$a_2 = 2$	$P_2 = 0 \times 2 + 1 = 1$	$Q_2 = 2$	$\frac{P_2}{Q_2} = \frac{1}{2}$
$a_3 = 1$	$P_3 = 1 \times 1 + 0 = 1$	$Q_3 = 1 \times 2 + 1 = 3$	$\frac{P_3}{Q_3} = \frac{1}{3}$
$a_4 = 2$	$P_4 = 2 \times 1 + 1 = 3$	$Q_4 = 2 \times 3 + 2 = 8$	$\frac{P_4}{Q_4} = \frac{3}{8}$
$a_5 = 2$	$P_5 = 2 \times 3 + 1 = 7$	$Q_5 = 2 \times 8 + 3 = 19$	$\frac{P_5}{Q_5} = \frac{7}{19}$
$a_6 = 1$	$P_6 = 1 \times 7 + 3 = 10$	$Q_6 = 1 \times 19 + 8 = 27$	$\frac{P_6}{Q_6} = \frac{10}{27}$
$a_7 = 3$	$P_7 = 3 \times 10 + 7 = 37$	$Q_7 = 3 \times 27 + 19 = 100$	$\frac{P_7}{Q_7} = \frac{37}{100}$

The approximating fraction $\frac{10}{27} = 3{\cdot}704$, which only differs from the actual fraction by 0·004 mm. per thread, may thus be accepted for all practical purposes.

$$\frac{10}{27} = \frac{2 \times 5}{3 \times 9} = \frac{20 \times 50}{45 \times 60}.$$

(*j*) *The Proof of the Sum.*

The comparison that 6·5 in. = 165 mm., or an adopted fraction, is not perfectly accurate. Should it be desired to find out to what extent the fraction which has been arrived at, and, consequently, the thread to be cut, deviate, this can

be done, when a metric thread has to be cut on a lathe having an English leadscrew, by multiplying the numerator of the fraction by the pitch of the leadscrew in mm. The product thus obtained should coincide with the product of the denominator of the fraction and the pitch to be cut, i.e. numerator × pitch of leadscrew in mm. = denominator × pitch of thread to be cut.

Numerator, denominator and leadscrew pitch being known, the pitch of the thread to be cut can consequently be determined.

On page 28 the fraction $\frac{26}{165}$ has been determined for a pitch to be cut of 2 mm., and a leadscrew of 2 threads per inch.

The product of numerator and leadscrew pitch in mm. is thus $26 \times 12{\cdot}69975$ or $26 \times 12{\cdot}7 = 330{\cdot}2$. This product when divided by the denominator of the fraction will give the pitch in mm. to be cut with the wheels determined on, thus, $330{\cdot}2 : 165 = 2{\cdot}001$ mm. The pitch is consequently exact to within 0·001 mm.

$$\frac{7 \times 13}{11 \times 20} = \frac{91}{220}$$

is given on page 29 for a pitch of $10\frac{1}{2}$ mm., with a leadscrew of 1 in. pitch.

$$\frac{91}{220} \times 25{\cdot}4 = \frac{2311{\cdot}4}{220} = 10{\cdot}5063 \text{ mm.}$$

The pitch is therefore exact to within 0·0063 mm. Both these differences may practically be regarded as of no consequence.

In the case of a lathe having a metric leadscrew on which English thread is to be cut, the denominator should be multiplied by 2·54. The numerator when divided by the product thus obtained, gives the pitch to be cut in inches.

On page 24, the fraction for cutting 6 threads per inch with a leadscrew of 10 mm. pitch is given as $\frac{165}{6 \times 65}$.

If the denominator be multiplied by 2·54, the result will be $\frac{165}{6 \times 65 \times 2{\cdot}54} = \frac{165}{990{\cdot}6}$.

Each pitch cut is thus 0·1665656 in.

The exact pitch = $\frac{1}{6}$ in. = 0·16̇ in., so that the thread cut differs only by 0·0001010 in.

Note, that when cutting metric thread with an English leadscrew, the thread cut is a fraction too coarse, whilst, on the contrary, when cutting English thread with a metric leadscrew, the thread obtained is a fraction too fine.

(*k*) *Fixing up the Wheels.*

It is not always possible to fix up the 4 wheels in the order of sequence given in the examples.

FIG. 10.

FIG. 11.

The following fraction may, for example, occur: $\frac{50 \times 30}{125 \times 55}$, in which case the wheels must be placed as per Fig. 10, although the wheels 30 and 55 cannot mesh.

The fraction can, however, be arranged in another order of

sequence, viz.: $\frac{30 \times 50}{55 \times 125}$, which makes fixing up possible (see Fig. 11), but care must be taken that the wheels of the numerator are never placed in the denominator, or *vice versâ*.

Should simple changing about of the factors in numerator and denominator, or one of them, be impossible, the fraction is then resolved into the lowest possible factors, and another combination of wheels sought for, which will give the same proportion between numerator and denominator, as, for example:

$$\frac{30 \times 50}{55 \times 125} = \frac{2 \times 2 \times 3 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 11} = \frac{30 \times 40}{55 \times 100} \text{ or } \frac{30 \times 40}{50 \times 110}.$$

(*l*) *Thread-cutting with Double Compound Train.*

Should it be necessary to cut a thread considerably coarser or finer than that of the leadscrew, it can easily happen that the necessary wheels are lacking.

For example, to cut 56 threads per inch, leadscrew 2 threads per inch.

The fraction is $\frac{2}{56} = \frac{10 \times 15}{70 \times 120}$. A wheel with 10 teeth is lacking. If the numerator and denominator of the fraction are once again multiplied by 2, a wheel with 140 teeth is obtained in the denominator, which is also not at hand.

In such a case, the numerator and denominator of the fraction are resolved into 3 factors, as, for instance:

$$\frac{2}{56} = \frac{20}{560} = \frac{2 \times 2 \times 5}{5 \times 8 \times 14} = \frac{20 \times 25 \times 30}{70 \times 75 \times 80}.$$

Example: To cut 48 threads per inch. Lead-screw 2 threads per inch.

Solution: $\frac{2}{48} = \frac{20}{480} = \frac{2 \times 2 \times 5}{5 \times 8 \times 12} = \frac{20 \times 25 \times 30}{60 \times 75 \times 80}$.

(*m*) *The Cutting of Left-hand Threads.*

So far, it has been implicitly taken for granted that only right-hand threads had to be cut; it can, however, happen, though not often, that a left-hand thread has to be cut. For this purpose, the leadscrew must rotate in an opposite direction to the lathe-spindle. This is obtained by connecting up an idle wheel at will. In double transmission, a fifth wheel (idle), chosen at will, may also be introduced.

A number of lathes have been constructed of late which render the connecting-up of an intermediate wheel unnecessary. With these lathes, all that is required is to shift the reverse-plate at the headstock which reverses the movement of the pinions which drive the change-wheels, thus causing these wheels and the leadscrew to rotate in an opposite direction. This is a decided improvement, as there is not much space to spare when five or six wheels are on the shear. With a double compound train generally the larger number are only small wheels, but with four wheels, however, every proportion is possible, so that the placing of a fifth wheel can sometimes be very troublesome.

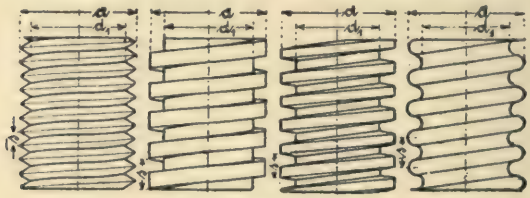
CHAPTER III.

THREADS AND THEIR CONSTRUCTION.

(*a*) *Forms of Thread.*

THERE are different forms of thread, a few of which are illustrated in Figs. 12–15.

Fig. 12 shows the Vee thread in its general form, which is constructed in different types, and is most often met with. Fig. 13 illustrates the square or flat thread, the section of which is either a square or a right-angle, and which is much in use for larger diameters and coarser threads. In Fig. 14, the trapezium thread is seen, the section of which is a trape-



FIGS. 12, 13, 14, 15.

zium, much in vogue for the leadscrews of lathes, the worm being also a trapezium thread. Fig. 15 is the round thread, formed by the intersection of semicircles.

Very little need be said with reference to the last three types, for which it is impossible to speak of any one system, the form of the section being dependent on circumstances, and determined by each individual at will.

Different varieties, however, exist of the Vee thread.

(*b*) *Types of Threads.*

The type chiefly employed is certainly the Whitworth system; Fig. 16 shows the construction.

The depth of the Whitworth thread is equal to 0·64 of

the pitch, the sides of the thread forming an angle of 55° with top and bottom rounded through $\frac{1}{6}$ of the line *h*, drawn perpendicular from the apex of the triangle to its base, the radius of rounding being equivalent to 0·143 *h*.

Not only is the sectional form of the Whitworth thread definitely fixed, but also the number of threads per inch for

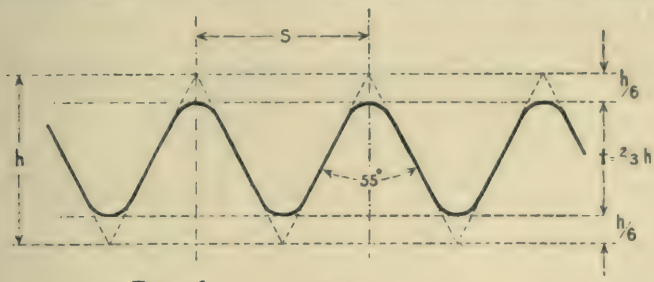


FIG. 16. h = 0,96. S

all diameters up to and including 6 inches, and this has been fixed at from 20–2¼ threads per inch.

The sectional form is precisely similar for the finest as well as the coarsest threads, and it is for this reason that the exact dimensions and strength of the thread are determined by the simple determination of the outside diameter.

TABLE I.—WHITWORTH THREAD.

Diameter at Bottom.		Diameter of Thread.		No. of Threads per inch.	Diameter at Bottom.		Diameter of Thread.		No. of Threads per inch.
in.	mm.	in.	mm.		in.	mm.	in.	mm.	
¼	6·35	·18	4·72	20	1⅜	34·92	1·16	29·46	6
5/16	7·94	·24	6·09	18	1½	38·1	1·29	32·68	6
⅜	9·52	·29	7·36	16	1⅝	41·27	1·37	35·28	5
7/16	11·11	·34	8·64	14	1¾	44·45	1·49	37·84	5
½	12·70	·39	9·91	12	1⅞	47·62	1·59	40·38	4½
⅝	15·87	·51	12·92	11	2	50·82	1·71	43·43	4½
¾	19·05	·62	15·74	10	2¼	57·15	1·93	49·02	4
⅞	22·22	·73	18·54	9	2½	63·5	2·18	55·37	4
1	25·4	·84	21·33	8	2¾	69·85	2·38	60·45	3½
1⅛	28·57	·94	23·87	7	3	76·2	2·63	66·80	3½
1¼	31·75	1·07	26·92	7					

Table I. gives the various dimensions of the Whitworth thread.

A Whitworth thread of certain dimensions can also be cut on a considerably larger outside diameter, the exact strength of the thread being fixed by simply determining which dimension of the Whitworth system is required.

Table I. gives not only the outside diameter, but also the diameter at bottom of thread, so that the height of the thread can be arrived at by subtracting the latter from the former, and dividing the difference by two.

When cutting threads on the lathe, which deviate in diameter from this system, it is necessary to know the depth of the thread both for cutting inside and outside threads.

The depth of the thread can also be arrived at by a simple calculation.

For this purpose, just look at Fig. 16. By drawing a perpendicular from the apex of the triangle, a right-angled triangle is formed, the smallest angle of which is equal to $55^\circ \div 2 = 27^\circ\ 30'$.

Tang. $27^\circ\ 30' = 0{\cdot}52$. Therefore, if the long side of the right-angle = 1, then the short side = $0{\cdot}52$, and the base of the triangle of $55^\circ = 1{\cdot}04$.

This base is, however, equal to S, i.e. the pitch.

Whence it follows that $h : S = 1 : 1{\cdot}04$, or $0{\cdot}96 : 1$.

The real depth of the thread is, however, only $\frac{2}{3}\,h$. So that the ratio between the depth of the thread and the pitch is equal to $\frac{2}{3}\,h : S = (0{\cdot}96 \times \frac{2}{3}) : 1 = 0{\cdot}64 : 1$. $\frac{2}{3}\,h$ thus equals $0{\cdot}64$ S.

If we take the outside diameter D, the diameter at the bottom of the thread d, and the pitch S, then, $d = D - 2 \times 0{\cdot}64$ S, or $d = D - 1{\cdot}28$ S.

The gas thread universally adopted by the pipe trade, given in Table II., is also according to the Whitworth system, and in 1903 was also adopted as the standard thread for pipes and fittings for gas, water, and steam by the Association of German Engineers, the Association of German Plumbers, the Association of the German Central Heating Industry, and the Union of German Pipe Manufacturers.

On the other hand, in the autumn of 1898, an attempt was made by a number of influential associations of Continental engineers, assembled in congress at Zurich, and including, amongst others, the Association of German Engineers, the

TABLE II.—WHITWORTH SCREWING THREAD.

Nominal Internal Diameter of Pipe.		External Diameter of Pipe.		Diameter at Bottom of Thread.		No. of Threads per in.	Nominal Internal Diameter of Pipe.		External Diameter of Pipe.		Diameter at Bottom of Thread.		No. of Threads per in.
in.	mm.	in.	mm.	in.	mm.		in.	mm.	in.	mm.	in.	mm.	
1/8	3·17	·382	9·71	·336	8·55	28	1½	38·1	1·882	47·81	1·765	44·85	11
¼	6·35	·518	13·15	·451	11·44	19	1⅝	41·27	2·02	51·33	1·904	48·37	11
⅜	9·52	·656	16·67	·589	14·95	19	1¾	44·45	2·047	52	1·93	49·03	11
½	12·7	·826	20·97	·734	18·64	14	2	50·8	2·347	59·61	2·23	56·65	11
⅝	15·87	·902	22·91	·81	20·59	14	2¼	57·15	2·587	65·72	2·47	62·76	11
¾	19·05	1·04	26·44	·949	24·11	14	2½	63·5	3·	76·23	2·882	73·27	11
⅞	22·22	1·089	30·2	1·097	27·87	14	2¾	69·85	3·247	82·47	3·13	79·51	11
1	25·4	1·309	33·24	1·192	30·28	11	3	76·2	3·485	88·51	3·368	85·51	11
1⅛	28·57	1·492	37·89	1·375	34·93	11	3½	88·9	3·912	99·36	3·795	96·39	11
1¼	31·75	1·65	41·91	1·533	38·95	11	4	101·6	4·339	100·2	4·223	107·26	11
1⅜	34·92	1·745	44·32	1·628	41·36	11							

TABLE III.—S. I. THREAD.

Diam.	Pitch.	Diameter at Bottom of Thread.	Diam.	Pitch.	Diameter at Bottom of Thread.	Diam.	Pitch.	Diameter at Bottom of Thread.
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
6	1	4·7	20	2·5	16·75	48	5	41·5
7	1	5·7	22	2·5	18·75	52	5	45·5
8	1·25	6·37	24	3	20·1	56	5·5	48·85
9	1·25	7·37	27	3	23·1	60	5·5	52·85
10	1·50	8·05	30	3·5	25·45	64	6	56·02
11	1·50	9·05	33	3·5	28·45	68	6	60·02
12	1·75	9·72	36	4	30·8	72	6·5	63·55
14	2	11·4	39	4	33·8	76	6·5	67·55
16	2	13·4	42	4·5	36·15	80	7	70·09
18	2·5	14·75	45	4·5	39·15			

Swiss Association of Machine-Tool Makers, the Society for the Encouragement of National Industries, etc., to replace the Whitworth system, which is based on the English system of measurements, by a metric thread, and it was unanimously decided to adopt the S. I. thread ("Système International"), as per Table III.

Owing to the universal application of the Whitworth thread, the innovation makes but little headway, though, especially of late years, this system is being more and more used on the Continent, especially by the Automobile Industry, for threads cut on the lathe.

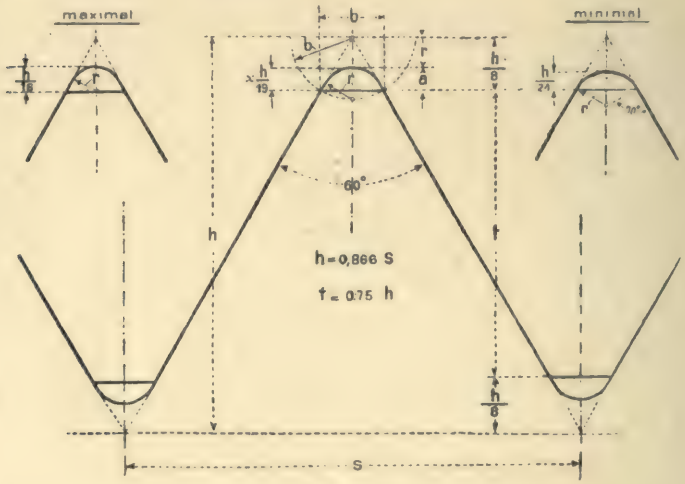


FIG. 17.

The construction and form of the S. I. thread is given in Figs. 17 and 18.

The apex is an angle of 60°. The section is consequently an equilateral triangle.

Hence it follows that the perpendicular h, dropped from the apex to the base, is equivalent to

$$\sqrt{\left(S^2 - \left(\frac{S}{2}\right)^2\right)} = h = 0{\cdot}866\,S.$$

The truncation equals $\frac{1}{8}\,h$, so that the thread has a height of $0{\cdot}75\,h$, or $0{\cdot}6495$ S.

Whilst the Whitworth thread bears not only at the sides but also at the bottom, the S. I. thread, on the contrary, has a play at the bottom of, at the most, $\frac{1}{16}\ h$, equivalent to the half truncation, the rounding of the thread is equal to the

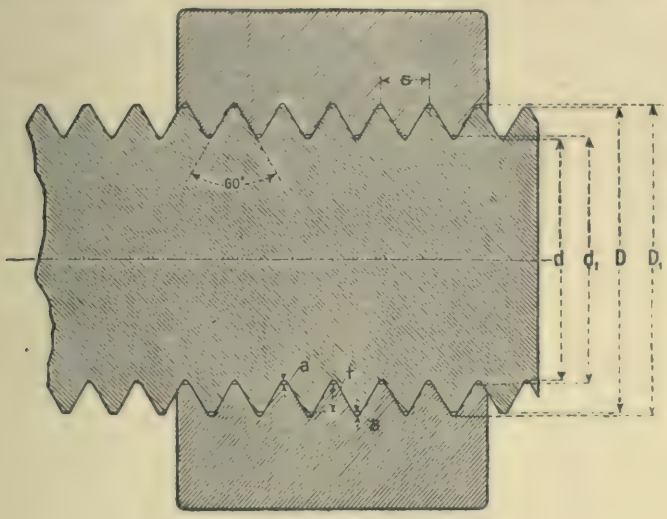


FIG. 18.

play, the radius of the rounding in this case being $\frac{1}{16}\ h$. The rounding and play amount, as is generally accepted, to at least $\frac{1}{24}\ h$. Loewe strikes an average for this, and fixes the play and rounding at $\frac{1}{19}\ h$.

The outside diameter of the male-screw is thus smaller than the diameter at bottom of the thread in the nut, and

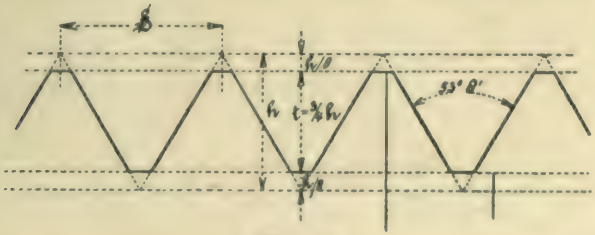


FIG. 19.

vice versâ, the diameter at bottom of the thread of the male-screw is smaller than the outside diameter of thread in the nut.

If we take the play a, then the actual depth of the thread of both male-screw and nut equals $0{\cdot}75\,h + a$. If we fix the play at its maximum, equals $\frac{1}{16}\,h$, then the height equals $0{\cdot}0625\,h + 0{\cdot}75\,h = 0{\cdot}8125\,h$, or $0{\cdot}703625\,S = \sim 0{\cdot}7\,S$.

The Löwenherz thread (Table IV.) is in general use up to

TABLE IV.—LÖWENHERZ THREAD.

Diam.	Pitch.	Diameter at Bottom of Thread.	Diam.	Pitch.	Diameter at Bottom of Thread.	Diam.	Pitch.	Diameter at Bottom of Thread.
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
1	0·25	0·625	2·6	0·45	1·925	5·5	0·9	4·15
1·2	0·25	0·825	3	0·5	2·25	6	1	4·5
1·4	0·3	0·95	3·5	0·6	2·6	7	1·1	5·35
1·7	0·35	1·175	4	0·7	2·95	8	1·2	6·2
2	0·4	1·4	4·5	0·75	3·375	9	1·3	7·05
2·3	0·4	1·7	5	0·8	3·8	10	1·4	7·9

a diameter of 10 mm. for instruments of every description, especially in Germany and Switzerland, and in screw works, the screws are almost exclusively made by this system.

The construction of the Löwenherz thread is shown in Fig. 19. The apex is 53° 8′.

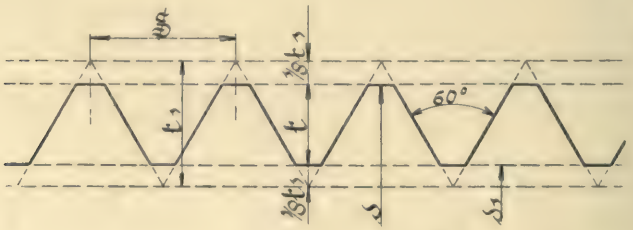


FIG. 20.

This angle results from $h = S$. The thread is truncated flat on the outside diameter and at bottom with a $\frac{1}{8}$ truncation, so that the real depth of the thread is $= 0{\cdot}75\,h$.

The Sellers thread (Table V.) is an American thread, constructed as per Fig. 20.

TABLE V.—SELLERS THREAD.

Diameter. inch.	Number of Threads per inch.	Diameter. inch.	Number of Threads per inch.	Diameter. inch.	Number of Threads per inch.
$\frac{1}{8}$	40	$1\frac{1}{8}$	7	$3\frac{1}{4}$	$3\frac{1}{2}$
$\frac{3}{16}$	24	$1\frac{1}{4}$	7	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{1}{4}$	20	$1\frac{3}{8}$	6	$3\frac{3}{4}$	3
$\frac{5}{16}$	18	$1\frac{1}{2}$	6	4	3
$\frac{3}{8}$	16	$1\frac{5}{8}$	$5\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{7}{8}$
$\frac{7}{16}$	14	$1\frac{3}{4}$	5	$4\frac{1}{2}$	$2\frac{3}{4}$
$\frac{1}{2}$	13	$1\frac{7}{8}$	5	$4\frac{3}{4}$	$2\frac{5}{8}$
$\frac{9}{16}$	12	2	$4\frac{1}{2}$	5	$2\frac{1}{2}$
$\frac{5}{8}$	11	$2\frac{1}{4}$	$4\frac{1}{2}$	$5\frac{1}{4}$	$2\frac{1}{2}$
$\frac{3}{4}$	10	$2\frac{1}{2}$	4	$5\frac{1}{2}$	$2\frac{3}{8}$
$\frac{7}{8}$	9	$2\frac{3}{4}$	4	$5\frac{3}{4}$	$2\frac{3}{8}$
1	8	3	$3\frac{1}{2}$	6	$2\frac{1}{4}$

The apex is an angle of 60°, so that the perpendicular t_1 dropped from the apex to the base, is again $= 0{\cdot}866\,S$. The thread is flat-faced at bottom and on the top with $\frac{1}{8}$ truncation, consequently

$$t = \tfrac{3}{4}\,t_1, \text{ and } 0{\cdot}75 \times 0{\cdot}866 = 0{\cdot}6495\ S.$$

The thread which resembles the S. I. thread very much has, however, no play and is divided according to English measurements.

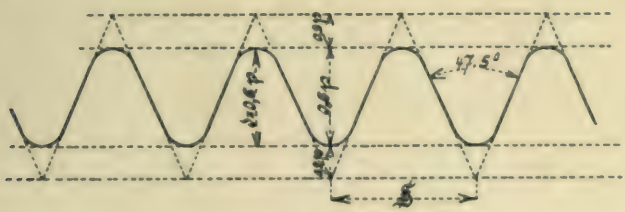


FIG. 21.

Although largely displaced by the Sellers thread, the sharp V thread still exists and is used in America. (See Table VI.) The section is an equilateral triangle not truncated.

The B. A. S. (British Association Standard), as per Table VII., is an English thread much used in England for screws of small diameter, especially for electric fittings. The apex, Fig. 21, is an angle of 47½°. The thread is truncated, and

TABLE VI.—SHARP V THREAD.

Diameter.	Number of Threads per inch.	Diameter.	Number of Threads per inch.	Diameter.	Number of Threads per inch.	Diameter.	Number of Threads per inch.
inch.		inch.		inch.		inch.	
¼	20	1⅛	10	1¾	5	2⅞	4
5/16	18	⅞	9	1⅞	4½	3	3½
⅜	16	15/16	9	2	4½	3⅛	3½
7/16	14	1	8	2⅛	4½	3¼	3½
½	12	1⅛	7	2¼	4½	3⅜	3¼
9/16	12	1¼	7	2⅜	4½	3½	3¼
⅝	11	1⅜	6	2½	4	3⅝	3¼
11/16	11	1½	6	2⅝	4	3¾	3
¾	10	1⅝	5	2¾	4	3⅞	3
						4	3

top and bottom are rounded, leaving the depth of the thread equal to 0·6 S.

In addition to the foregoing, the Delisle, Sauvage, Acme, and Thury systems are to be met with.

The total number of thread systems exceeds fifty, but only the seven most used have been treated of here.

TABLE VII.—B.A.S. THREAD.

Number	0	1	2	3	4	5	6	7	8	9	10	12	14	19
Diameter. mm.	6	5·3	4·7	4·1	3·64	3·2	2·8	2·5	2·2	1·9	1·7	1·3	1	0·79
Pitch. mm.	1	0·9	0·81	0·73	0·66	0·59	0·53	0·48	0·43	0·39	0·35	0·28	0·23	0·19

(*c*) *Screw-cutting Tools.*

A tool used for screw-cutting must first and foremost be perfectly true. It is not to be looked upon as an ordinary tool, nor may it be ground on a stone which does not run true.

When cutting deep threads, whether they be V or square, it is always advisable to use separate tools for roughing and finishing.

The cutting angle must be about 70°, whilst the tool must not be pointed or semi-circular, but flattened at the edge (Figs. 22 and 23), as otherwise the angle will not be true, and,

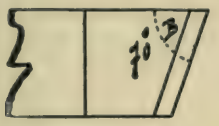


FIG. 22.

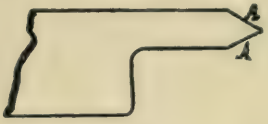


FIG. 23.

at the same time, it will be impossible to grind the tool accurately. The tool must not only stand on its edge in the angle B, Fig. 22, but the sides A A must also have clearance. The angle in which the thread lies on the work has also to be taken into consideration, and the line A B, Fig. 24, must run

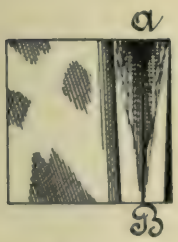


FIG. 24.

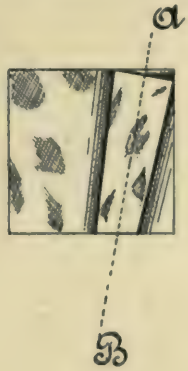


FIG. 25.

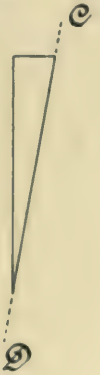


FIG. 26.

at the same angle. Suppose that a 1 in. pitch has to be cut on a diameter of 2 in. Then, imagine C D, in Fig. 26, to be the angle at which the thread lies on the work, the line A B of

the tool, Fig. 25, must thus run parallel to the line C D in Fig. 26. This is still more evident in the case of square threads with a coarse pitch, Fig. 27. In this case, the clearance

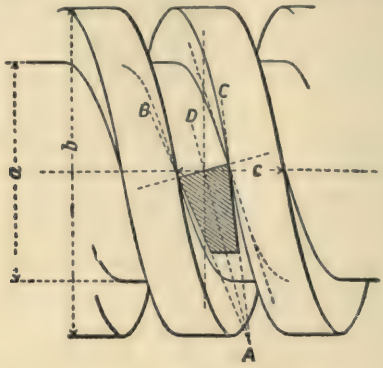
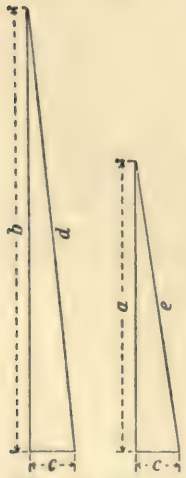


FIG. 27.

on the sides of the tool must be different. The diameter of the thread on the top, as also the angle of the thread there is indicated in Fig. 28, that at bottom of the thread in Fig. 29, *a* and *b* being the circumference, *c* and *c* the pitch, which is the same for both, and there are consequently two angles. The hypotenuses *d* and *e* show the angle of the thread at top and bottom. If the clearance of the tool is correct on the top, it will be incorrect when at the bottom. The steeper the pitch, the more noticeable this will be. The tool must have more clearance on the right-hand side for bottom than at the top, but less on the left-hand side. The tool must consequently be ground in such a manner that the right-hand side will have enough clearance at bottom of the thread, whilst the clearance for the left-hand side must concur with the angle at the top, that is to say, for a right-hand thread, as in Fig. 27 ; for left-hand threads or for internal threads the opposite conditions will exist in regard to angles. The tool must accordingly be



FIGS. 28 AND 29.

ground as indicated in Fig. 27, A B being the slope of the right-hand side of the tool, A C on the left-hand side. The upper cutting surface of the tool must run square on the line A D. When cutting an inside right-hand thread, everything is reversed, what is right-handed becoming left.

For a Vee thread, the tool must be ground in accordance with the angle of the system of the thread. It need scarcely be said that this must not be left only to eye or the rough estimate of the operator. A gauge should be provided, as

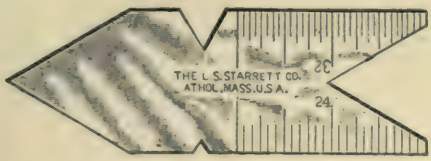


FIG. 30.

per Fig. 30, giving the precise angle. And yet, notwithstanding that it is far more difficult for a workman to judge an *angle* with the eye than to guess a certain *length*, and no one would ever think of permitting an operator to estimate a certain length without using his rule, it is an exception when the operator is provided with a suitable angle gauge.

It is utterly impossible that a thread can be true when the operator has judged the angle of the tool with his naked eye.

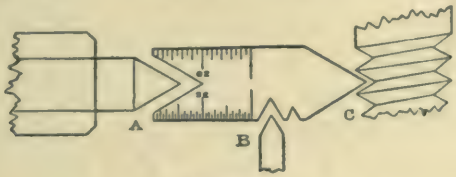


FIG. 31.

This gauge fulfils a second, and not less important, purpose. Even though the tool be ground to the precise angle, it is still possible to cut a wrong thread, for the tool must be so placed in the holder that an imaginary line drawn perpendicularly from the apex of the triangle to the imaginary base, must also fall perpendicularly on the side of the cylinder on which the thread is to be cut. Not having this gauge,

the operator judges with his eye the position in which he thinks the tool should be placed. But the most experienced workman can make a mistake, *it is not possible for the thread to be true.* If the tool has been placed with the utmost care in the position which might reasonably be supposed to be correct, and this is afterwards checked with an angle gauge,

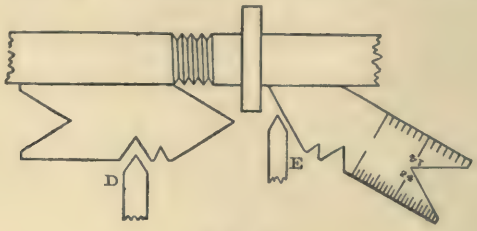


FIG. 32.

it will almost invariably be found that the position is incorrect. The reason is that the two lines forming the angle are usually very short in proportion to the other lines of the tool, the eyes being consequently deceived.

In Fig. 31, at A, is shown the manner of gauging the angle to which a lathe centre should be turned; at B, the angle to

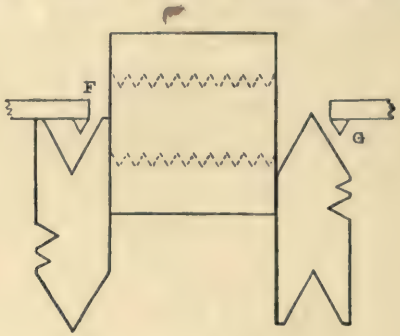


FIG. 33.

which a screw thread-cutting tool should be ground; and at C, the correctness of the angle of a screw thread already cut.

In Fig. 32, the shaft with a screw thread is supposed to be held between the centres of a lathe. By applying the gauge as shown at D or E, the thread tool can be set at right angles

to the shaft, and then fastened in place by the bolts in the tool post, thereby avoiding imperfect or leaning threads.

In Fig. 33, at F and G, the manner of setting the tool for cutting internal threads is illustrated.

(*d*) *Cutting the Thread.*

As previously stated, it is always advisable to begin cutting a thread that has anything like a deep curve with a roughing tool which is at a cutting point and which need not be ground precisely to the angle.

The thread should afterwards be gone over with a finishing tool. When engaged in cutting shallow threads, the tool can cut on both sides at the same time, and it can be put exactly on the direction of the shaft. With deeper threads, i.e. quick pitches, this is no longer possible. Cutting with both sides of the tool at the same time causes it to snap, the thread is rough, and very often it is impossible to continue working; the tool should, therefore, work but one side at a time, should frequently be set slightly in a parallel direction to that of the shaft, and directly there is any play between the tool and the thread, it must again be set square on the direction of the shaft. Each time that the tool has gone completely over the thread, it should be withdrawn and again set in the original position at the commencement, though increased with the amount cut at one passage.

For this purpose a graduated collar is provided to the feed screw by means of which the traverse movement can be read, and by which the tool can be set in the exact position every time.

The operator formerly got out of the difficulty by marking the position of the screw spindle with a piece of chalk.

On lathes of up-to-date construction, the graduated collar is now always to be found on the screw spindle.

A very practical construction is shown in Fig. 34. Advantage is here taken of the movement of the two half-nuts when opening and closing, to withdraw the cutting tool from the curve, and *vice versâ*, back again to the exact

position, so that instead of having to carry out various operations at the end of the thread, a simple movement of a handle is all that is required.

The construction is as follows: Over the two half-nuts which move under the carriage in the same direction as the cross-slide, and are opened and closed by a double right- and left-hand screw, is placed a ⊓-shaped slide fixed on knobs of the upper portion of the half-nuts. The screw spindle of the cross-slide fits in the upper portion of this slide on the one side by a turned up edge, and on the other by lock-nuts. The screw spindle must consequently follow the movement of the slide. Holes are drilled right through the projecting

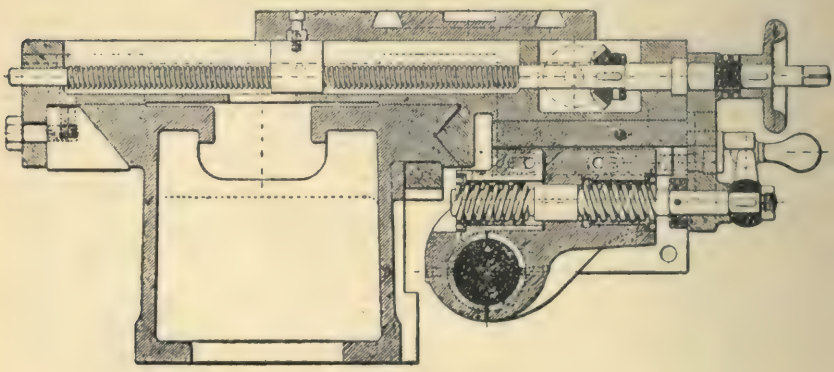


FIG. 34.

parts of the half-nuts, and the slide. A steel pin fits closely into these holes. Oblong holes, in which the pin has play, are bored in the carriage for same.

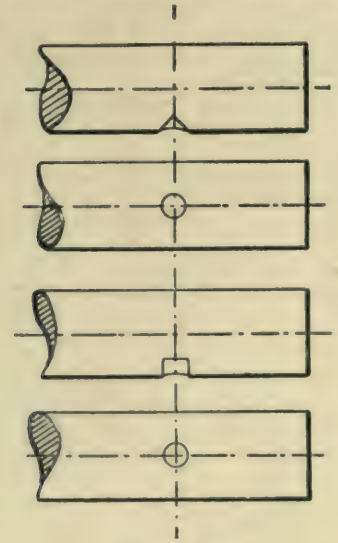
Before beginning to cut internal or external threads, the pin is set in the foremost or hindmost nut, so that the half-nut through which the pin is placed is coupled with the slide in which the screw spindle fits, and consequently they must follow the movement of the half-nut in question together with the cross-slide and tool. It is worked as follows: As soon as the tool has arrived at the end of the thread, the half-nuts of the lead-screw are opened and by this means the tool is withdrawn from the thread. The carriage is then

returned by hand by means of the pinion, the tool set so much farther in with the screw spindle as it is desired to cut deeper, and the half-nuts are closed again. This causes the tool to resume its original position, only cutting the material so much deeper as it has been set farther in by hand. If no

FIG. 35.

thread is to be cut, the connection between the slide and half-nut is broken by withdrawing the locking-pin, and the slide is coupled to the cross-slide by inserting the pin in the hole bored through both slides.

When screw cutting, this arrangement results in a decided



FIGS. 36–39.

saving of time, besides preventing the possibility of mistakes arising from inserting the tool either too far or not far enough in.

There should be an outlet for the tool at the end of the

thread. If the diameter is sufficiently large to permit of it, an entire circular groove should be turned, Fig. 35. If, for some reason or other, a circular groove is not possible, a suitable outlet, as per Figs. 36–39, must be drilled for vee or square threads. Before commencing cutting, the tool should be so fixed that it will arrive just at these holes.

It was formerly the custom to return the carriage when the tool had gone over the thread, by reversing the movement of the lathe. But with the present-day construction of the lathe, by which it is possible to return the carriage quickly by hand by means of the handle, the half-nuts are opened and the carriage returned by hand. If the thread being cut is of the same pitch as, or an aliquot part of the pitch of the leadscrew, the half-nuts can be dropped into engagement at any point of the leadscrew without any difficulty, the tool always returning to its precise position in the thread. This is, however, not so when the number of threads per inch are uneven or broken, and other means must be adopted to ensure the tool returning to its precise position in the groove. Consequently, when starting to cut the thread, a stop, or marking line, is placed on the bed, the half-nut closed and a chalk line drawn on top of the leadscrew, and another chalk line at the front side of the chuck-plate. When the tool has gone over the thread and the carriage has been returned by hand as far as the stop or the line, the head spindle is turned round till both chalk lines are again in their original position, the nuts closed, and the tool is once more in its precise place in the path which has just been cut.

This comparatively troublesome and primitive manner of working is done away with, if the carriage is provided with a thread indicator as shown in Fig. 40.

The following is the principle of this attachment: A small worm-wheel runs on the leadscrew, and by means of a pinion gearing, causes an indicator to move on a circular index-plate. All that is now necessary is to note the position of the indicator at the starting point, after which, the half-nuts can be closed, and the tool will come precisely in the path each time the indicator resumes its original position.

(*e*) *The Cutting of Double or Multiple Threaded Screws.*

The cutting of double or multiple threaded screws causes a good deal of trouble, as, in addition to exercising ordinary care that the thread cut is true, another most important point has to be taken into consideration, viz. that the setting of the tool is also exactly equidistant. The manner of working is similar to that for a single thread, but care should be taken as far as possible that when cutting a double thread the spindle wheel is divisible by two, and for a treble thread by three.

After the first incision has been made to the required depth, the tool must be shifted exactly to the centre between

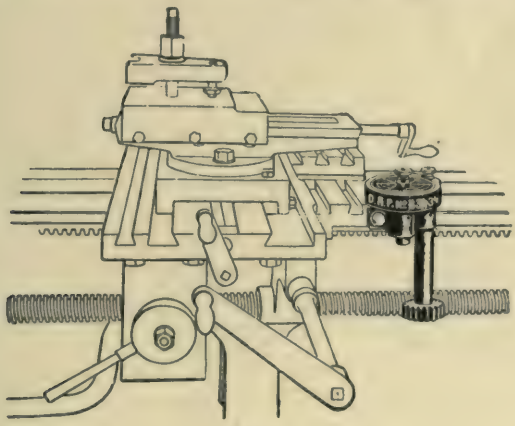


FIG. 40.

two threads for a double, and to one-third of the intermediate space for a treble thread. The distance the tool is to be shifted should, however, never be measured off, as this can never be exact, but must be obtained by mechanical means, either by turning the work-piece while the leadscrew is stationary, or by turning the leadscrew while the work-piece remains stationary. If a double thread has to be cut, one of the teeth of the spindle wheel coming between two teeth is marked with chalk, as also the two teeth which the tooth in question engages. After this the spindle wheel is bisected and this tooth is also chalked ; the spindle wheel is then released

from the wheel it engages, the spindle is given half a turn by hand, so that the opposite tooth comes between the two marked teeth, and the two wheels are once more engaged. If the spindle wheel is not divisible by two, then this must be found on the wheel on the leadscrew, but the pitch of the thread to be cut must in this case be taken into consideration.

For example.—A double threaded screw of 4 threads per 3 inches is to be cut on a lathe with a leadscrew of 2 threads per inch.

$$\text{The fraction is } \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2} = \frac{75}{50}.$$

The spindle wheel is, however, not divisible by 2, and as the factor 3, which is indivisible by 2, will invariably be found in that wheel, 4 wheels are used so that the factor 3 can be placed in the intermediate wheel.

$$\frac{75}{50} = \frac{100 \times 60}{50 \times 80}.$$

If there is any reason, for instance, with heavy lathes not to turn the spindle but to shift the carriage by turning the leadscrew, this is accomplished as follows for the above example :—

Pitch = $\frac{3}{4}$ in. The carriage must thus be shifted $\frac{3}{4} \div 2 = \frac{3}{8}$ in. ; the leadscrew has a pitch of $\frac{1}{2}$ in., and so must make $\frac{3}{8} \div \frac{1}{2} = \frac{3}{4}$ revolution ; the wheel of the leadscrew has 80 teeth, and consequently $80 \times \frac{3}{4} = 60$ teeth must be moved.

If the same pitch is to be cut on this lathe but for a three-thread, then the first-mentioned wheels, $\frac{75}{50}$ are the best to use ; the wheel with 75 teeth can be divided into three, and 25 teeth turned each time.

If it is desired to move the carriage, this must be moved $\frac{3}{4} \div 3 = \frac{1}{4}$ in., the leadscrew make $\frac{1}{2}$ revolution, and the wheel with 50 teeth be moved $50 \times \frac{1}{2} = 25$ teeth.

For example.—To cut a pitch of $1\frac{7}{8}$ in. Double threaded screw. Leadscrew 1 in. pitch.

Solution : $$\frac{1\frac{7}{8}}{1} = \frac{15}{8} = \frac{100 \times 60}{40 \times 80}.$$

For a double threaded screw, the spindle wheel is divisible by 2.

$1\frac{7}{8} \div 2 = 1\frac{5}{16}$ in. The leadscrew must thus make $1\frac{5}{16} \div 1 = 1\frac{5}{16}$ revolution.

$1\frac{5}{16} \times 80 = 75$. The wheel on the leadscrew must thus be moved 75 teeth.

Example.—To cut 6 threads per 15 in., three-threaded screw. Leadscrew $\frac{1}{2}$ inch pitch.

Solution : $$\frac{1\frac{5}{16}}{\frac{1}{2}} = \frac{15 \times 2}{6} = \frac{75 \times 80}{30 \times 40}.$$

For a three-thread, the spindle wheel can be divided into 3×25 teeth.

The carriage must be shifted $1\frac{5}{6} \div 3 = \frac{5}{6}$ in., so that the leadscrew must make $\frac{5}{6} \div \frac{1}{2} = 1\frac{0}{6}$ revolutions.

The wheel with 30 teeth is placed on the leadscrew, and $30 \times 1\frac{0}{6} = 50$ teeth are moved $= 50 \div 30 = 1$ revolution and 20 teeth.

(*f*) *The Cutting of very Coarse Thread.*

When cutting coarse thread, a difficulty may possibly occur which will require careful consideration. When the thread to be cut is considerably coarser than that of the leadscrew, the movement of the leadscrew must be appreciably quickened. There is, however, a limit to this, and that is the resistance offered by the teeth of the gear-wheels. If the pitch is too coarse, these will break off. The extent to which the pitch may be increased depends, naturally, entirely on the strength of the wheels supplied with the lathe. Generally speaking, the pitch may safely be a four-fold of the leadscrew, anything exceeding this being attended with considerable danger and the off-chance of the teeth breaking.

In order to permit thread to be cut which is many times coarser than that of the leadscrew, a gearing can be attached to the fast headstock, as illustrated in Fig. 2.

The wheel 15 can be set in connection with the small gear-wheel of the double back gearing. If then the lathe runs with

double back gear, the ratio of speed between the cone-pulley and lathe spindle will be 1 : 8, that between the wheel 15 and the cone-pulley pinion 2 : 1, and the wheel 10 will complete 4 revolutions to 1 of the spindle. In the case of a thread which is four times coarser than the leadscrew, there is a ratio

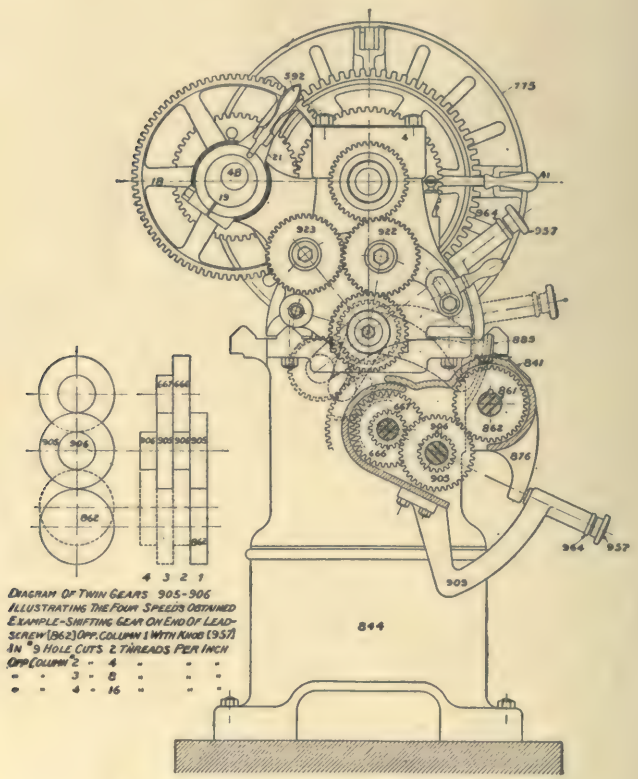


FIG. 41.

of 1 : 1 between the change-wheels, whilst for a thread eight times as coarse, there is only a ratio of 1 : 2. Taking as above that the teeth are strong enough for a ratio of 1 : 4, and that the leadscrew has a $\frac{1}{2}$ in. pitch, then a $4 \times 4 \times \frac{1}{2} = 8$ in. pitch may easily be cut in this manner.

(*g*) *The Hendey-Norton System.*

One of the newest designs for screw-cutting is that of the Hendey-Norton system, which, by means of a train of gears placed under and at the side of the headstock, renders it possible to cut a number of threads of different pitches

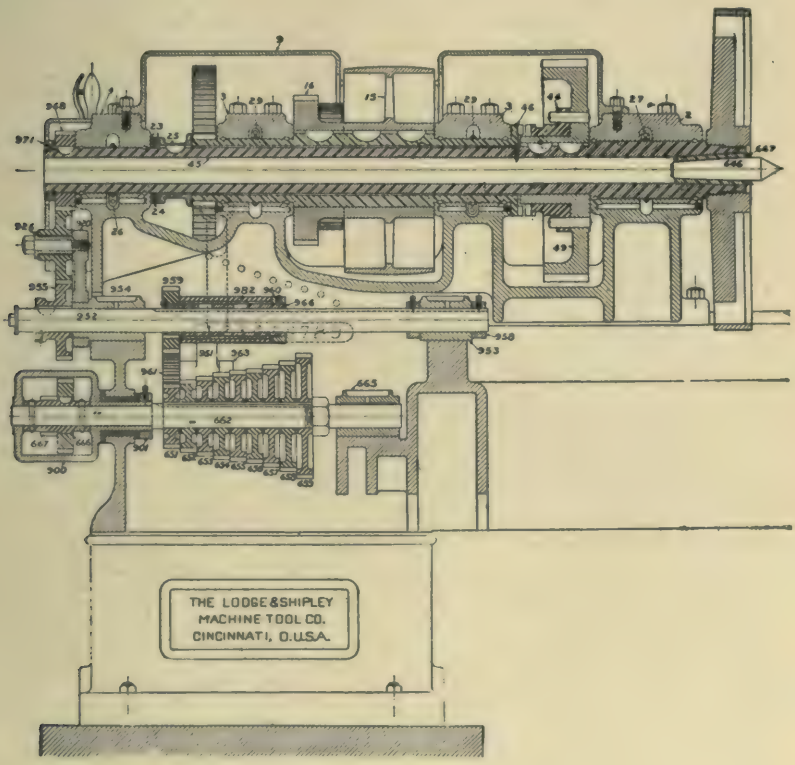


FIG. 42.

without the necessity of fixing different change-wheels. Change-wheels, as they have up till now been understood in connection with the lathe, have been entirely superseded. On a lathe provided with the Hendey-Norton system, it is no longer necessary to fix up or take off change-wheels, the various wheels being simply and solely geared up in the space

formed between the spindle and the leadscrew by the shifting of handles. The calculation of change-wheels is consequently a thing of the past.

But, in this work which treats of the whole question of screw-cutting in an abridged form, a description of this system, which will certainly come more and more to the front in the struggle for economical tools, and has already been very largely adopted, must not be missing.

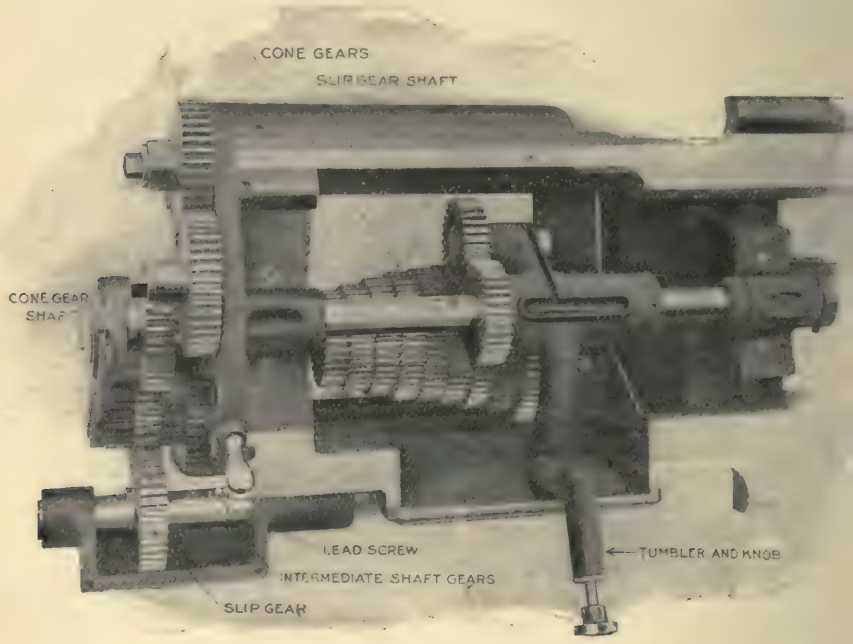


FIG. 43.

Arrangement of wheels in a Lodge and Shipley lathe, the fast headstock being removed.

On a lathe of this description, screw-cutting has been reduced to its simplest possible form. A clever workman may, it is true, be quite capable of calculating the wheels required to cut a certain thread quickly, and can possibly reckon it out in his head, but even so, the actual fixing up of the wheels seriously interferes with the steady progress of

the work, whilst the difficulty is at once doubled whenever turning, drilling, and thread-cutting have to be performed periodically, as, with so many lathes, the attendant circumstances are such that it cannot be arranged for all at the same time.

The lathes under discussion are constructed in such a manner that a great variety of threads can be cut without requiring the fixing up or taking off of a single wheel.

In the earlier constructions of this type of lathe, there was invariably one great drawback, viz. that the number of pitches which could be cut was comparatively small (10–12 pitches), but this number has now been extended to from 40 to 44 different pitches.

The foregoing illustration (Fig. 43) shows the complete arrangement of the wheels.

This gives a clear view of the bed, the fast headstock having been removed for the purpose.

The arrangement of the wheels consists of two separate groups of wheels. The first group (9–11 wheels) is placed under the headstock, the second being in a closed box attached to one side of the lathe.

The action performed by a workman in gearing up the wheels for the cutting of different pitches is extremely simple, so that after a brief explanation it is sufficiently clear even to a novice, and it can be executed so quickly that not more than from 10–20 seconds are required to change the wheels for another pitch than that for which they were geared up.

An index plate is affixed to the gear-box, which is given on page 80 in its exact size.

A handle with pointer is placed under the plate. This pointer can be moved over the entire length of the index plate and set in the middle of either of the four divisions of the plate. This handle is connected with the wheel indicated in Fig. 41, by the number 862, which accordingly moves this wheel with it; whilst under the holes in the headstock the numbers 1–8 or 1–11 appear, according to the dimensions of the lathe.

If, for example, it is required to cut $5\frac{3}{4}$ threads per inch

Thds.	Knob.	Thds.	Knob.	Thds.	Knob.	Thds.	Knob.
18	2	9	2	4½	2	2	1
19	3	9½	3	4¾	3	2¼	2
20	4	10	4	5	4	2½	4
22	5	11	5	5½	5	2¾	5
23	6	11½	6	5¾	6	2⅞	6
24	7	12	7	6	7	3	7
26	8	13	8	6½	8	3¼	8
28	9	14	9	7	9	3½	9
30	10	15	10	7½	10	3¾	10
32	11	16	11	16	11	4	11
FEEDS.							
80 to 40		40 to 20		20 to 10		10 to 5	

INDEX PLATE.

the pointer is placed by means of the handle in the middle of that division in which the number in question appears under the letters Thds. (Threads), in this particular case, in the second division on the right hand side. On the same line on which the number 5¾ appears, the figure 6 is to be found. The handle on the headstock is now placed in the hole above the figure 6, and the wheels are then geared up for cutting the desired thread. For all other threads appearing on the index plate, the procedure is identical. The topmost handle 957 is placed in the highest or lowest position, according as it is desired to cut left or right hand thread.

We will now proceed to give a detailed description of the construction of this gearing.

Wheel 968 (see Fig. 42) is fast on the lathe spindle and engages wheel 922 (Fig. 41) whenever right-hand thread is to becut. In this case wheel 923 is idle. For a left-hand thread, wheel 968 engages 923, and wheel 922 is caused to rotate by wheel 923, so that the direction of movement is just the reverse to that in the first case. Both wheels run loose on studs fastened in plate 920, and are shifted by the middle

handle. Wheel 922 engages wheel 955 which is fixed on shaft 952, which is consequently brought into motion. This same shaft 952 imparts motion to wheel 959, which, by means of a keyway, can be moved in a transverse direction by the handle under the fast headstock. Wheel 959 engages 961, which can be geared up, by means of the handle already referred to, with all the different wheels 651–659 under the fast headstock, which wheels are all fixed on shaft 662 ; wheel 961 consequently imparting motion to the shaft. Wheels 666 and 667 are also keyed to shaft 662. Wheel 862 (Fig. 41) movable by a keyway, is mounted on the leadscrew. Consequently the motion of shaft 662, to which the gear-wheels are keyed, is transmitted to wheel 862 by one of the wheels 666 or 667, viâ two sets of double wheels 905 and 906, both of which sets are identical.

This train of gears can be seen in the detailed drawing, Fig. 41, to the left of the side view of the fast headstock. It should be noted that wheels 905 and 906 are coupled, but that each set is independent of the other, and can consequently rotate at different speeds ; this is, moreover, apparent with the whole train of gears, seeing that, whilst wheels 666 and 667 also coupled, and each engages one of the sets 905 and 906, the latter obtain various speeds. This train of gears gives four different speeds between shaft 662 and the leadscrew.

Wheel 666 engages 905 and 906 on the right. Wheel 667 engages 905 and 906 on the left.

By moving wheel 862 on the leadscrew (this wheel is also to be seen in the illustration, Fig. 43), and by changing handle 964, which turns on shaft 662 and to which at the same time the two sets of wheels 905 and 906 are keyed, wheel 862 can be placed in four different positions, 1, 2, 3, and 4. (See detailed drawing, Fig. 41.)

Wheel 667 = 906 and wheel 666 = 905 = 862. The proportion of 667 to 906 = 1 : 1, of 666 to 906 = 2 : 1, so that if wheel 862 engages 905 on the right, the speed of shaft 662 is doubled, seeing that $\frac{2 \times 2}{1 \times 2} = 2$.

If wheel 862 engages 906 on the right, the motion of the

shaft is transmitted without any variation, and wheel 906 on the right simply serves as an idle wheel. If 862 engages 905 to the left, there is a double reduction in speed ; if 862 engages 906 on the left, the diminution is four times as great. Consequently, if the handle on the fast headstock is set in opening No. 9:

With the pointer in column 1, $3\frac{1}{2}$ pitches per inch
" " 2, 7 " "
" " 3, 14 " "
" " 4, 28 " "
} will be cut.

In this manner, with 11 wheels on shaft 662, 44 different pitches can be cut.

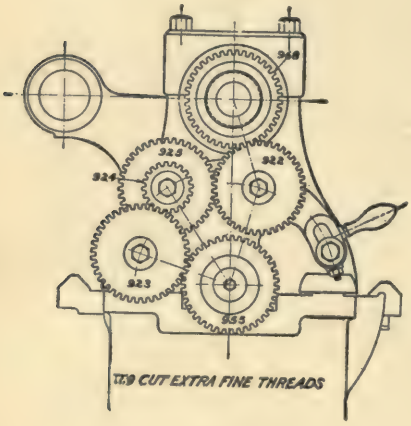


FIG. 44.

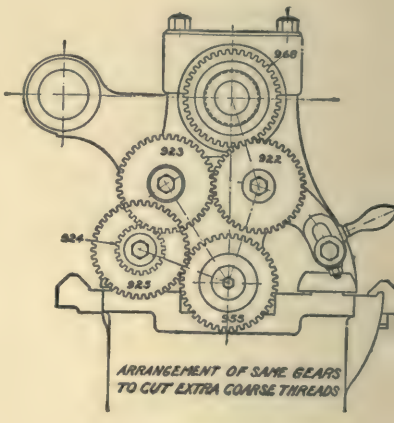


FIG. 45.

The swing plate of the fast headstock is further so constructed that, by setting up one wheel, the speed of the lead-screw can once more be doubled, or by removing the same wheel, it can be reduced to half as slow again, so that all the threads appearing on the index table can now be cut, with double or half the number per inch. The reserve hole in the swing plate can be clearly seen in Fig. 41, close to 923.

In the foregoing illustrations, Fig. 44 gives the combination for fine threads, Fig. 45 for coarse threads, whilst Fig. 46 shows the position of the wheel 955.

The usual gearing is: Wheel 968 engages 922, and 922 engages 955, the wheel on shaft 952. For fine threads, 968 engages 922, and 922 engages 923, consequently 923 engages 924, which is a double wheel with 925, the proportion between them being 1 : 2. Finally, 923 engages 955. Wheel 955 does not engage 922, but is moved a wheel's width to one side. (See Fig. 46.)

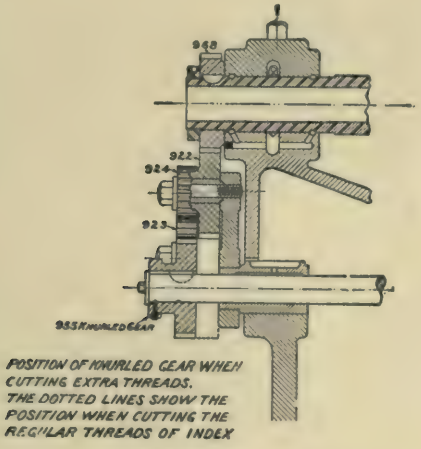


FIG. 46.

For coarse threads, 968 engages 922, and 922 engages 925, consequently 924 engages 923 and 923 engages 955. From a careful consideration of these two combinations for fine and coarse threads, it will be seen that wheels 924 and 925 on the one side, and wheel 923 on the other side, are mutually interchanged for the two cases.

So far it has only been multiples or fractions of an inch, or both, which could be cut in this manner. Should it, however, be necessary to deviate herefrom for any special pitch, other threads than those of the English system can also be cut by a certain proportion between the two wheels 924 and 925.

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